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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

MOMENTS IN CONTINUOUS RECTANGULAR SLABS ON RIGID SUPPORTS

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Synopsis

A method of analysis for rectangular slabs, supported on four sides and continuous over the supports, is described in this paper. The solution is so arranged that the equations providing continuity between the panels can be expressed in terms of the restraining moment at the middle of each support. The computations are simplified by using a sine distribution for the edge moments and by establishing continuity between the panels only at the middle of the supports. These approximations enable the solution to be based on assumptions that more nearly represent the fundamental structural action than does the use of hypothetical beam strips. Diagrams are provided from which the equations can be expressed easily. These equations are solved most conveniently by successive approximations.

Introduction

The moments, shears, and deflections in a single rectangular panel, either simply supported or fixed at the edges, have been computed for many types of loading. When a floor slab is continuous over the supports in all directions, an exact mathematical solution is possible but time-consuming, as the behavior in any panel then varies with the loading and physical characteristics of contiguous panels. Along with this action, the vertical deflection and torsional stiffness of the supporting beams provide additional disturbing factors. Confronted with these difficulties, the engineers who have developed the various building codes have naturally emphasized the most obvious action of the slab—that is, the flexure of the member in its two principal directions.[3] When the slab is simply supported, this flexural action is apparent, and the selection of

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 15, 1941.**

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[3] "Building Regulations for Reinforced Concrete," *Proceedings*, A. C. I., 1936, p. 181.

two orthogonal strips, such as AB and CD, Fig. 1(*a*), that are considered as simply supported beams is a natural approximation. The distribution of the applied loading to these strips is an indeterminate problem that is frequently solved by making the deflections of the strips equal at their intersection. Ordinarily such assumptions give safe results for the maximum bending moments at the center of the slab, as the torsional resistance of the slab is entirely neglected. By referring to Fig. 1(*b*), it can be seen that the twisting moments T_3 and T_4 on an element of the slab can assist in the transfer of the vertical load to the support along the strip CD and thereby reduce the bending moments M_1 and M_2. In the same manner, the twisting moments T_1 and T_2, Fig. 1(*c*), can reduce the bending moments M_3 and M_4 in any strip AB.

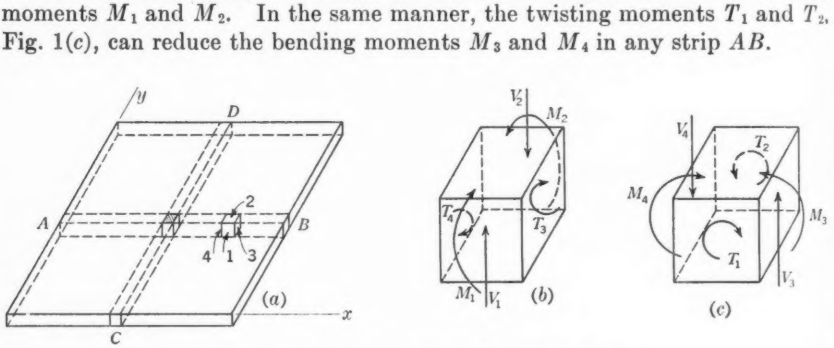


FIG. 1.—COUPLES ACTING ON AN ELEMENT OF A SLAB

In some building codes, such as the German Building Regulations, this reduction in the bending moments due to the torsional stiffness of the slab is effected by multiplying the moments that are obtained from an analysis of the individual strips by a reduction factor (Marcus approximation) that has been obtained from a slab analysis.[4] The present American Concrete Institute (A. C. I.) building regulations[3] in the United States also adhere to the use of individual strips that are considered as continuous beams in the case of continuous floor slabs. In this case, the only correction for the slab action is made by an empirical distribution of the load between the two hypothetical beams. In such analyses, continuity of strain throughout the slab is provided as a secondary factor, if at all, rather than as a fundamental part of the solution. The only alternative to the strip method appears to be in the direction of using the entire panel as a structural unit.

The use of the entire panel was adopted by Nathan M. Newmark, Assoc. M. Am. Soc. C. E., in his analysis of slabs that are continuous in one direction and simply supported on the edges parallel to this direction.[5] In that problem, where only two edges of any panel are subjected to bending moments, numerical tabulation of edge moments and of the corrections for edge rotations is feasible and can be applied directly. On the other hand, when slabs are continuous in all directions, there is considerable advantage in an algebraic arrangement that enables the conditions of continuity to be expressed by equations of standard

[4] "Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten," by H. Marcus, Berlin, Springer, 1932, p. 89.

[5] "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," by Nathan M. Newmark, *Bulletin No. 304*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1938.

form. Successive approximations then can be used in the numerical solution of these equations. This procedure was used by Prof. S. Timoshenko in his solution of a rectangular panel with fixed edges.[6]

Regardless of whether an algebraic or direct arithmetical form of arrangement is used, the solution requires that the edge slopes of the elastic curve of the slab be expressed as the sum of the slope in a simply supported slab due to the applied loading, plus the slopes due to the restraining moments at the four edges. In the case of fixed edges, the resultant slope must be zero, but for continuous slabs the resultant slope in one panel must be equal and opposite to the resultant slope of the same edge in the adjacent panel. It should be noted that the foregoing procedure calls for satisfying the continuity conditions entirely around the boundaries of the various panels. Also, before the equations can be formulated, the edge slopes must be expressed in terms of the applied loads and the edge moments. This relationship is mathematically possible because of the fortunate circumstance that, for most types of loading, the slopes can be expressed as converging series of the type[7]

$$\theta_y = A_1 \sin \frac{\pi x}{a} + A_2 \sin \frac{2 \pi x}{a} + A_3 \sin \frac{3 \pi x}{a} + \cdots \qquad (1)$$

in which the coefficients can be evaluated in terms of the load and the ratio $\frac{b}{a}$, or $\frac{a}{b}$, which involves the dimensions of the slab. An exact solution is obtained if the first terms of each series are made to satisfy the required slope condition, then the second terms, then the third terms, etc. The actual edge moments will be the algebraic sum of the moments that satisfy the separate terms of the series. For instance, if the slopes along the boundary AD of the simply supported panel $ABCD$, Fig. 2(a), are considered, the rotation θ_y' at any distance x from the origin A, for a uniform load p per unit area over the entire panel, when $\frac{b}{a} = 1.0$, is equal to

$$\theta_y' = \frac{p\,a^3}{D}\left(0.0137 \sin \frac{\pi x}{a} + 0.00025 \sin \frac{3 \pi x}{a} + 0.000033 \sin \frac{5 \pi x}{a} + \cdots\right) \qquad (2a)$$

and, when $\frac{b}{a} = 1.5$:

$$\theta_y' = \frac{p\,a^3}{D}\left(0.0184 \sin \frac{\pi x}{a} + 0.00025 \sin \frac{3 \pi x}{a} + 0.000033 \sin \frac{5 \pi x}{a} + \cdots\right) \qquad (2b)$$

In all such equations the term D is used to represent the flexural resistance of

[6] "Bending of Rectangular Plates with Clamped Edges," by S. Timoshenko, *Proceedings*, Fifth International Cong. for Applied Mechanics, 1938.

[7] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1940, Chapters 5 and 6.

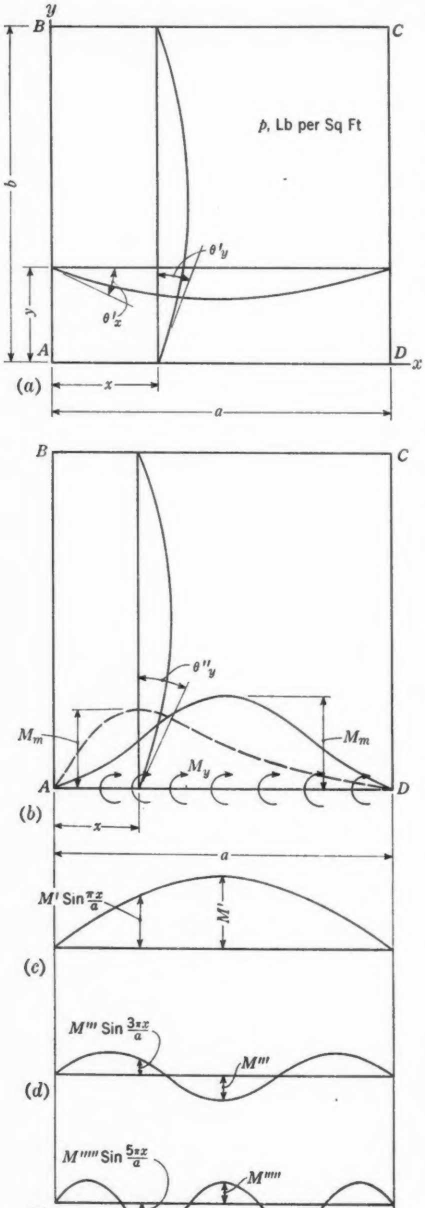


FIG. 2

the slab and is equal to

$$D = \frac{E\,h^3}{12\,(1 - \mu^2)} \quad \text{.......(3)}$$

in which h is the thickness of the slab and μ equals Poisson's ratio. The restraining moments along any boundary, such as AD, Fig. 2(b), can be represented by a corresponding sine series, as

$$M_y = M_1' \sin \frac{\pi x}{a} + M''' \sin \frac{3 \pi x}{a} + M''''' \sin \frac{5 \pi x}{a} + \cdots \quad \text{......(4)}$$

For this distribution of moments, the slopes along any edge also can be expressed as a sine series, thus, when $\frac{b}{a} = 1.0$:

$$\theta_y'' = \frac{a}{D}\left(0.156\,M' \sin \frac{\pi x}{a} + 0.053\,M''' \sin \frac{3 \pi x}{a} + 0.0318\,M''''' \sin \frac{5 \pi x}{a} + \cdots\right) \quad \text{.(5a)}$$

and, when $\frac{b}{a} = 1.5$:

$$\theta_y'' = \frac{a}{D}\left(0.159 \sin \frac{\pi x}{a} + 0.053\,M''' \sin \frac{3 \pi x}{a} + 0.0318\,M''''' \sin \frac{5 \pi x}{a} + \cdots\right) \quad \text{.(5b)}$$

If the edge AD is fixed while the others are simply supported, $\theta_y' + \theta_y''$ must be equal to zero for all points along the boundary AD. This condition can be realized if each term of the series for θ_y' is made equal but with opposite sign to the corresponding term of θ_y''. Thus, using Eqs. 2a and 5a, for θ_y' and θ_y'', when $\frac{b}{a} = 1.0$: For $\sin \frac{\pi x}{a}$,

$$\frac{0.156\,M'\,a}{D} = -\,\frac{0.0137\,p\,a^3}{D}; \qquad \text{or,} \qquad M' = -\,0.0877\,p\,a^2$$

for $\sin \frac{3\pi x}{a}$,

$$\frac{0.053\, M'''\, a}{D} = -\frac{0.00025\, p\, a^3}{D}; \qquad \text{or,} \qquad M''' = -0.0047\, p\, a^2$$

for $\sin \frac{5\pi x}{a}$,

$$\frac{0.0318\, M'''''\, a}{D} = -\frac{0.000033\, p\, a^3}{D}; \qquad \text{or,} \qquad M''''' = -0.0010\, p\, a^2$$

and the edge moments are given by the series:

$$M_y = -p\, a^2 \left(0.0877 \sin \frac{\pi x}{a} + 0.0047 \sin \frac{3\pi x}{a} + 0.001 \sin \frac{5\pi x}{a} + \cdots\right) \ldots\ldots (6a)$$

The moment M_a at the middle of the edge, when $x = \frac{a}{2}$, will be

$$M_a = -p\, a^2 (0.0877 - 0.0047 + 0.001 + \cdots) = -0.084\, p\, a^2 \ldots (6b)$$

In a similar manner, Eqs. 2*b* and 5*b*, the corresponding series for $\frac{b}{a} = 1.5$, give

$$M_y = -p\, a^2 \left(0.1158 \sin \frac{\pi x}{a} + 0.0047 \sin \frac{3\pi x}{a} + 0.001 \sin \frac{5\pi x}{a} + \cdots\right) \ldots\ldots (7)$$

and, when $x = \frac{a}{2}$, $M_a = -0.112\, p\, a^2$.

If the panels are restrained on all sides, the slopes on the four sides must be expressed in terms of all four restraining moments and the applied loading. This numerical procedure is not particularly difficult when only one panel is to be investigated, and it yields results that are mathematically correct. However, when continuity must be established between several panels, all of which may be irregular, and in view of the uncertainties of the assumptions, it seems reasonable to relinquish some mathematical exactness for more facility of application. Such an approximation, of course, should give results that are consistent with the accuracy obtained in the average design procedure. For this reason, consideration will now be given to an approximate solution which the writers believe will yield sufficiently accurate results for average conditions and which will be practicable in its application. Any unusual condition of loading, such as a heavy concentrated load near one corner, will require the more exact solution.

Approximate Method of Solution

The distribution of the restraining moments along any support of a continuous floor slab may be either asymmetrical, as shown by the dotted line for side AD, Fig. 2(*b*), or symmetrical, as shown by the solid line. For many practical conditions of loading, the maximum value M_m will not vary greatly from the value M' (Fig. 2(*c*)) for a distribution of $M' \sin \frac{\pi x}{a}$, although the actual distribution is somewhat different. The type of loading best suited for such an approximation will naturally be one that is fairly symmetrical about the center of the panel, such as a uniform load over the entire panel or concentrated loads near the center. Unusual conditions, such as heavy concentrated loads near a support, will ordinarily be critical with respect to shear and, therefore, will require special consideration.

To simplify the numerical solution further, continuity between the various panels will be maintained only at the middle of each edge instead of entirely around the boundaries of each panel, as for the exact method. For any continuous slab, such as that in Fig. 3, the edge moments can be expressed in terms

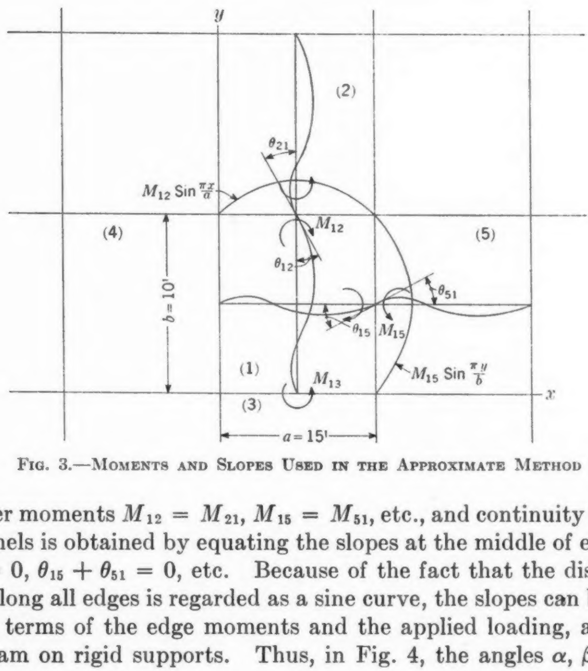


FIG. 3.—MOMENTS AND SLOPES USED IN THE APPROXIMATE METHOD

of the center moments $M_{12} = M_{21}$, $M_{15} = M_{51}$, etc., and continuity between the various panels is obtained by equating the slopes at the middle of each edge, as $\theta_{12} + \theta_{21} = 0$, $\theta_{15} + \theta_{51} = 0$, etc. Because of the fact that the distribution of moments along all edges is regarded as a sine curve, the slopes can be expressed directly in terms of the edge moments and the applied loading, as for a continuous beam on rigid supports. Thus, in Fig. 4, the angles α, β, and γ are given in terms of the moment $M' \sin \frac{\pi x}{a}$ for various values of $\frac{b}{a}$. Therefore, any edge slope such as θ_{12}, Fig. 3, can be expressed in the form

$$\theta_{12} = \alpha \left(\frac{M_{12}\, a}{D}\right) + \beta \left(\frac{M_{13}\, a}{D}\right) + \gamma \left[\frac{(M_{14} + M_{15})\, b}{D}\right] + \phi_s \ldots\ldots (8)$$

in which ϕ_s is the slope in a simply supported slab for the applied loading. Values of ϕ_s are given in Fig. 5(*a*) for a uniform load p per unit area over the entire panel, for a concentrated load P at the center of the panel, and, in Fig. 5(b), for a line load L across the center of the panel. It should be noted that, to determine the value of θ for a particular edge, the angles α and β are obtained by taking a as the length of that side, whereas for selecting γ the values of b and a are interchanged.

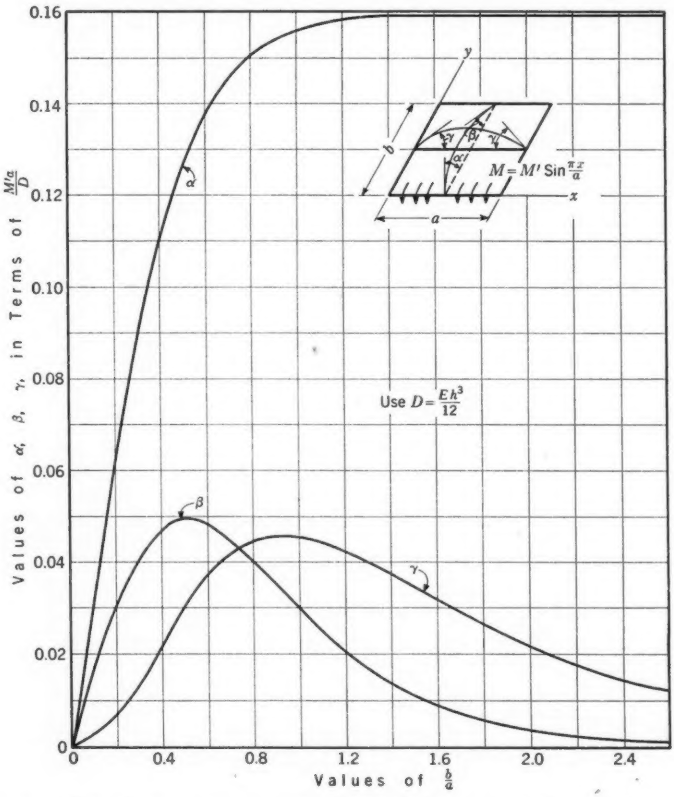


FIG. 4.—EDGE SLOPES α, β, AND γ IN A SIMPLY SUPPORTED SLAB DUE TO MOMENTS APPLIED ALONG ONE SIDE

To illustrate the use of the foregoing values, the equation for θ_{12} in panel (1), Fig. 3, will be given for a uniform load p over the entire area. For selecting α, β, and ϕ_s, $\frac{b}{a} = \frac{10}{15} = 0.67$; and, for selecting γ, $\frac{b}{a} = \frac{15}{10} = 1.5$. By Figs. 4 and 5(a):

$$\theta_{12} = 0.143\left(\frac{15\,M_{12}}{D}\right) + 0.046\left(\frac{15\,M_{13}}{D}\right) + 0.035 \times 10\left(\frac{M_{14} + M_{15}}{D}\right) + 0.0077 \times 15^3\,\frac{p}{D} \qquad (9)$$

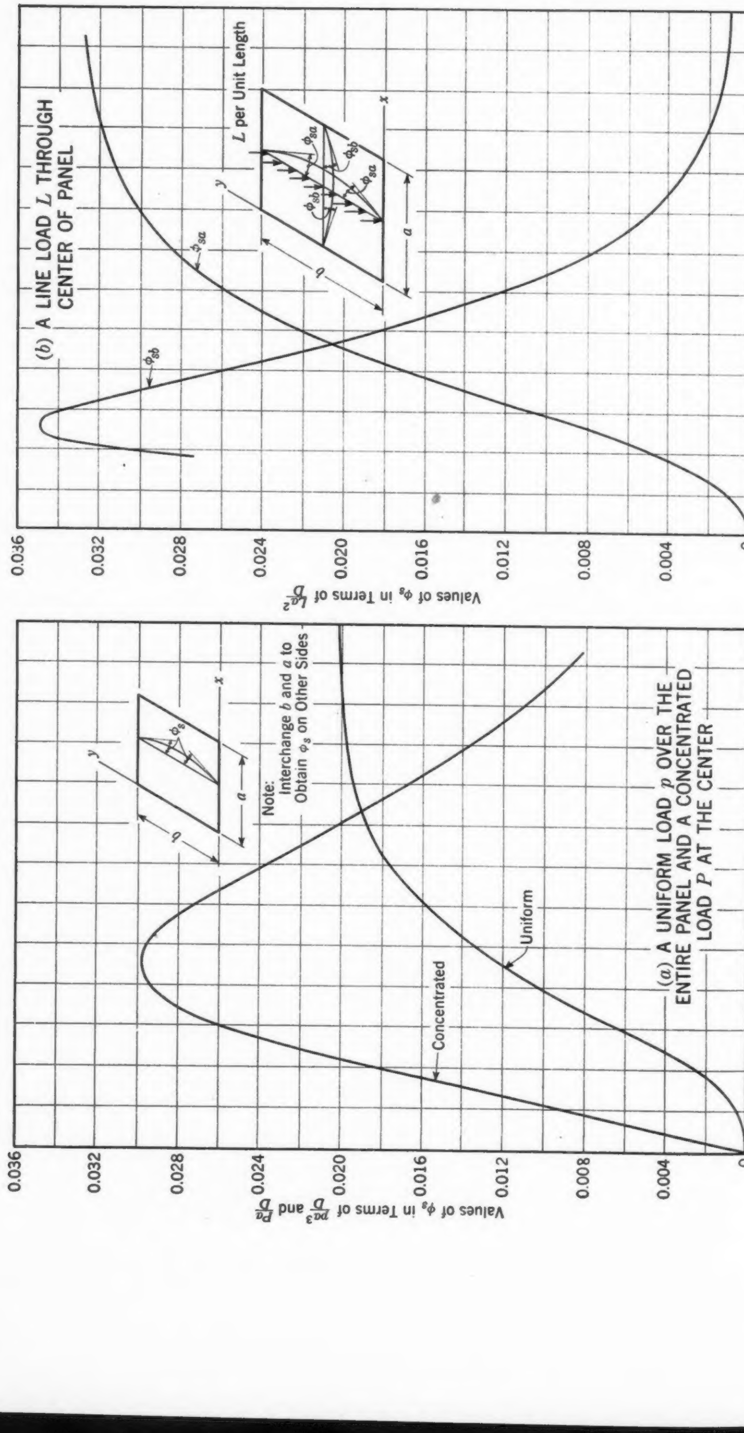


FIG. 5.—EDGE SLOPES ϕ_s

Other values of θ can be expressed by similar equations, and the edge moments can be determined from the continuity condition.

Numerical Example.—The moments in the continuous floor slab shown in Fig. 6(a) will be solved by the approximate method for a uniform load p over

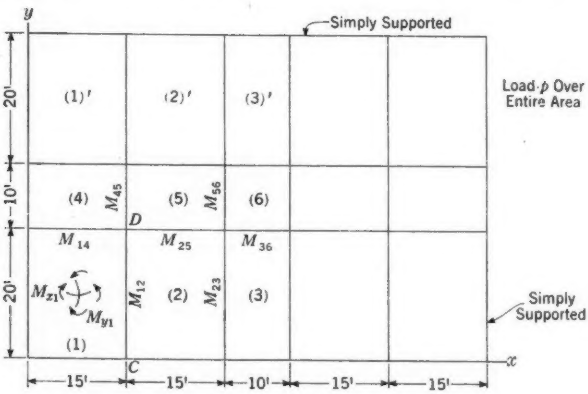


FIG. 6

the entire floor. Because of symmetry, only seven edge moments are required. The values of α, β, γ, and ϕ_s from Figs. 4 and 5(a) are listed in Table 1. The

TABLE 1.—VALUES OF α, β, γ, AND ϕ_s FOR FLOOR SLAB IN FIG. 6

Panel No.	$\frac{b}{a}$	α	β	γ	ϕ_s	$\frac{b}{a}$	α	β	γ	ϕ_s
1, 2	1.33	0.1583	0.0153	0.0395	0.0171	0.75	0.148	0.0425	0.0435	0.00925
3	0.5	0.126	0.049	0.0306	0.00425	2.0	0.159	0.003	0.0215	0.0198
4, 5	0.67	0.143	0.046	0.041	0.0077	1.50	0.159	0.011	0.035	0.0182
6	1.0	0.156	0.030	0.045	0.0134	....	....	....	....	....

value of $D = \frac{E\,h^3}{12}$ is constant for all panels. The effect of Poisson's ratio n will be considered later. For continuity between panels (1) and (2), the condition $\theta_{12} + \theta_{21} = 0$ must be satisfied:

$$\frac{a}{D} M_{12} (\alpha_{12} + \alpha_{21}) + \frac{a}{D} M_{23} \beta_{21}$$

$$+ (M_{14} + M_{25}) \gamma \left(\frac{b}{D}\right) + \phi_{s12} + \phi_{s21} = 0 \ldots\ldots\ldots\ldots (10)$$

Substituting the values from Table 1:

$$(0.148 + 0.148)\, 20 \frac{M_{12}}{D} + 0.0425 \left(\frac{20\, M_{23}}{D}\right)$$

$$+ 0.0395 \frac{15\,(M_{14} + M_{25})}{D} = - \frac{2 \times 0.00925\, p\, 20^3}{D};$$

or,

$$0.296\, M_{12} + 0.0425\, M_{23} + 0.0295\, (M_{14} + M_{25}) = -\, 7.40\, p \ldots (11a)$$

In a similar manner, the following equations can be obtained:

For $\theta_{23} + \theta_{32} = 0$—

$$0.0425\ M_{12} + 0.3239\ M_{23} + 0.0295\ M_{25} + 0.0107\ M_{36} = -\ 5.324\ p \ldots (11b)$$

for $\theta_{45} + \theta_{54} = 0$—

$$0.318\ M_{45} + 0.0107\ M_{56} + 0.1213\ (M_{14} + M_{25}) = -\ 3.64\ p \ldots (11c)$$

for $\theta_{56} + \theta_{65} = 0$—

$$0.0107\ M_{45} + 0.3448\ M_{56} + 0.1213\ M_{25} + 0.0914\ M_{36} = -\ 3.16\ p . (11d)$$

for $\theta_{14} + \theta_{41} = 0$—

$$0.3478\ M_{14} + 0.0580\ M_{12} + 0.0230\ M_{45} = -\ 5.55\ p \ldots (11e)$$

for $\theta_{25} + \theta_{52} = 0$—

$$0.3478\ M_{25} + 0.0580\ (M_{12} + M_{23}) + 0.0230\ (M_{45} + M_{56}) = -\ 5.55\ p \ldots (11f)$$

and for $\theta_{36} + \theta_{63} = 0$—

$$0.3449\ M_{36} + 0.1238\ M_{23} + 0.0914\ M_{56} = -\ 3.321\ p \ldots (11g)$$

Eqs. 11 can be solved conveniently with a slide rule by the method of iteration. If approximate values of the larger moments are first obtained, the entire set of moments can be tabulated quickly. The numerical values of these moments, together with the corresponding values of the same moments that are given by the exact method and by the A. C. I. Code, are given in Table 2.

TABLE 2—EDGE MOMENTS AT MIDDLE OF SUPPORTS, IN TERMS OF p, FOR FLOOR SLAB IN FIG. 6

Description	M_{12}	M_{23}	M_{45}	M_{56}	M_{14}	M_{25}	M_{36}
Approximate	−20.98	−12.64	−2.90	−4.64	−12.25	−9.83	−3.87
Exact	−20.56	−12.14	−3.95	−5.40	−12.17	−10.05	−4.26
A. C. I.	−13.18	−9.20	−3.54	−2.94	−8.18	−5.84	−2.80

The distribution of edge moments along the support CD between panels (1) and (2) (Fig. 6) is shown in Fig. 7 for the exact method, together with the dis-

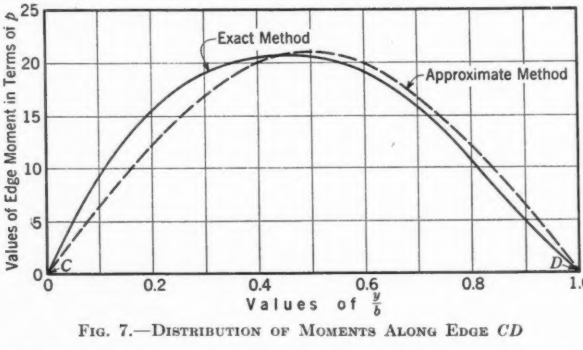


FIG. 7.—DISTRIBUTION OF MOMENTS ALONG EDGE CD

tribution that is obtained by the approximate method. It can be seen that the maximum ordinate for the exact distribution is near the center and is practically equal to the maximum ordinate of the assumed sine curve.

Moments at the Center of Panels

After the edge moments have been evaluated, the moments at the centers of the various panels can be computed by taking the algebraic sum of the center moments due to the applied loading on a simply supported panel and those caused by the restraining edge moments. Fig. 8(*a*) gives the center moments for a uniform load over the entire panel, Fig. 8(*b*) for a concentrated load at the center, and Fig. 8(*c*) for a line load through the center. The values for the concentrated load, which are taken from the solution[8] presented in 1930 by H. M. Westergaard, M. Am. Soc. C. E., are based on the assumption that it is distributed over a circular area whose diameter is 0.10 a, that Poisson's ratio is equal to 0.15, and that $\frac{a}{h} > 13$. The center moments that are caused by a distribution of moments alongside a of $M' \sin \frac{\pi x}{a}$ are given in Fig. 8(*d*) for various values of $\frac{b}{a}$. For the effects of moments along the adjacent sides, the values of a and b and the direction of x and y can be interchanged.

To illustrate the use of Fig. 8, the positive moments will be calculated at the center of panel (1), Fig. 6:

$$M_{x1} = 0.0618\, p\, 15^2 - 0.144 \times 20.98\, p - 0.132 \times 12.25\, p = 9.28\, p \quad (12a)$$

and

$$M_{y1} = 0.032\, p\, 15^2 - 0.1365 \times 20.98\, p - 0.0015 \times 12.25\, p = 4.36\, p \quad (12b)$$

A comparison of the center moments for all panels of the continuous slab in Fig. 6 is given in Table 3.

TABLE 3.—Comparison of Positive Moments at Center of Panels for Slab in Fig. 6, in Terms of p

Description	M_{x1}	M_{y1}	M_{x2}	M_{y2}	M_{x3}	M_{y3}	M_{x4}	M_{y4}	M_{x5}	M_{y5}	M_{x6}	M_{y6}
Approximate Method:												
$\mu = 0$	9.28	4.36	7.96	2.60	2.08	−0.98	−0.26	2.48	0.42	2.87	2.07	1.93
$\mu = 0.2$	10.15	6.22	8.48	4.19	1.88	−0.56	+0.24	2.43	0.99	2.95	2.46	2.34
Exact Method; $\mu = 0$	9.34	4.27	7.88	2.72	1.98	−0.76	−0.27	2.38	0.35	2.84	2.06	1.83
A. C. I.	7.21	6.16	5.94	4.13	2.30	1.55	1.72	−0.06	1.81	0.84	1.28	1.10

In the foregoing analysis, Poisson's ratio μ has been taken equal to zero. This assumption has little effect upon the edge moments, but the positive moments may be affected considerably. However, when the positive moments are known for $\mu = 0$, they can be computed for any other value of μ from the relations:

$$M_x = M_x' + \mu\, M_y' \quad (13a)$$

and

$$M_y = \mu\, M_x' + M_y' \quad (13b)$$

in which M_x' and M_y' are the values for $\mu = 0$. For instance, the positive

[8] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *Public Roads*, Bureau of Public Roads, U. S. Dept. of Agriculture, Vol. II, No. 1, 1930.

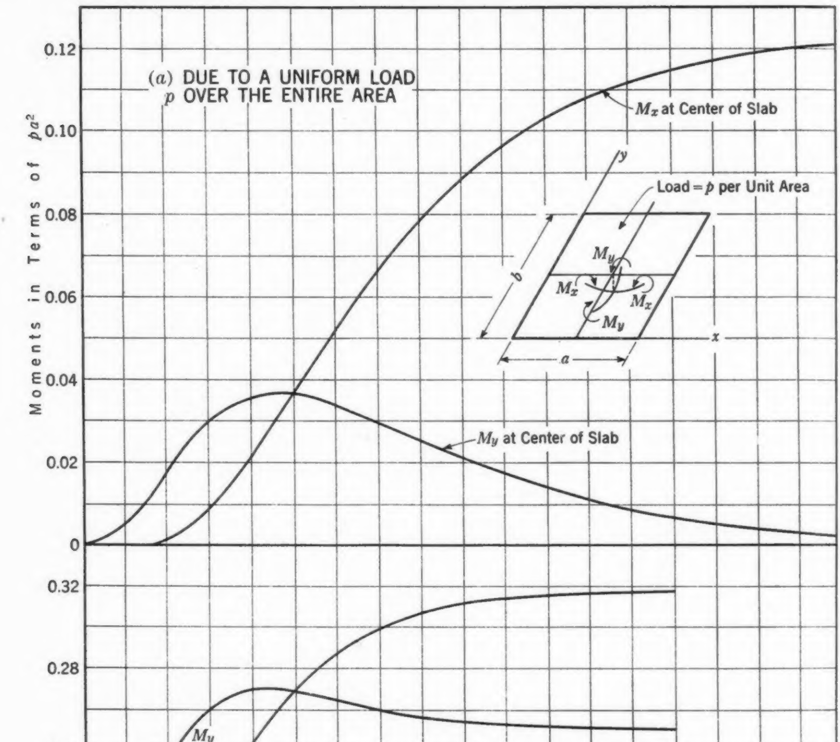
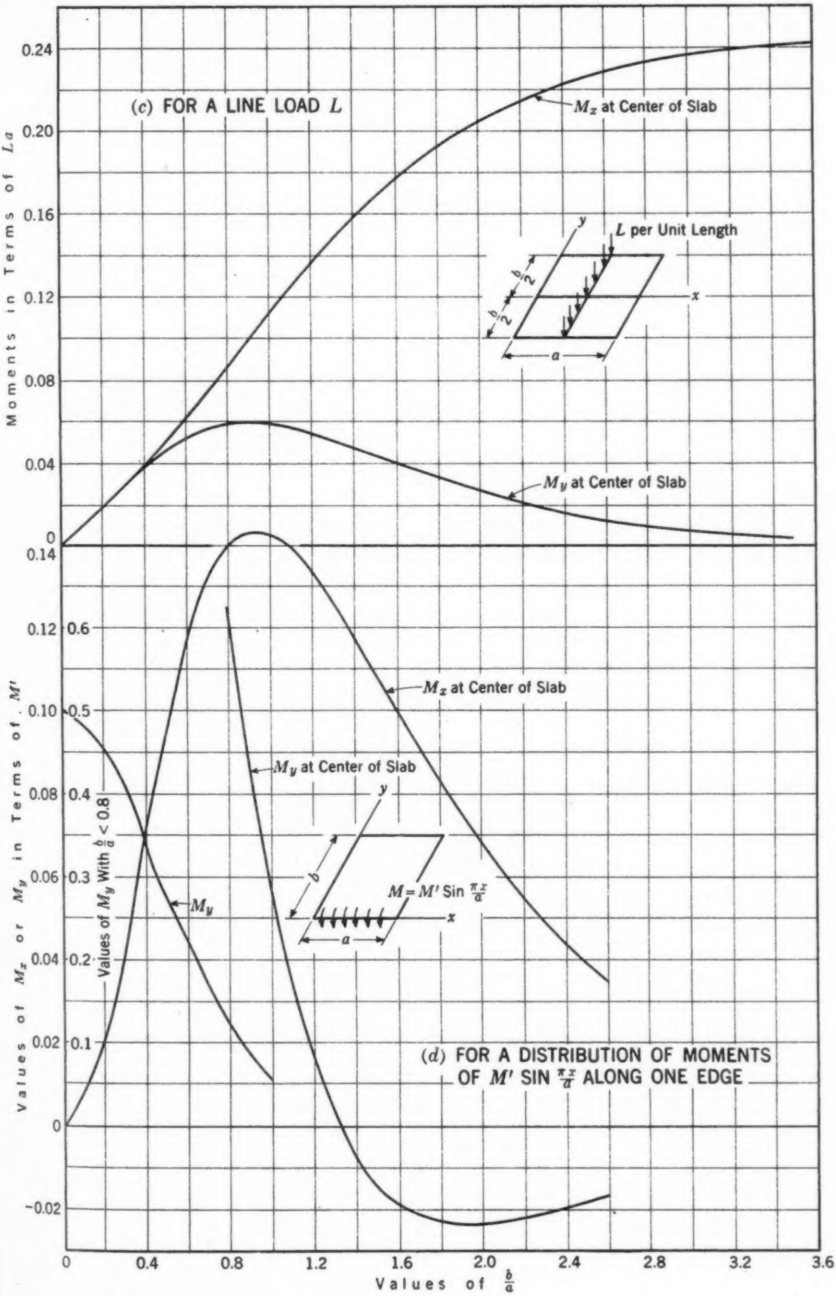


FIG. 8.—MOMENTS AT THE CENTER



OF A SIMPLY SUPPORTED SLAB

moments in panel (1), Fig. 6, for $\mu = 0.2$, are:

$$M_{x1} = 9.28\,p + 0.2 \times 4.36\,p = 10.15\,p \ldots\ldots\ldots\ldots (14a)$$

and

$$M_{y1} = 0.2 \times 9.28\,p + 4.36\,p = 6.22\,p \ldots\ldots\ldots\ldots (14b)$$

This change in the value of μ produces an increase in the center moments of 9.4% and 42.6%, which indicates that some consideration should be given to this factor. All diagrams in this paper are for $\mu = 0$, except Fig. 8(*b*), which is based on $\mu = 0.15$. If the moments M_x and M_y are known for any value of μ, the values of M_x' and M_y' for $\mu = 0$ can also be determined by Eqs. 13.

Conclusion

From the numerical examples that are given in the paper, together with many others that are not recorded herein, the following conclusions appear warranted:

(*a*) The solution of continuous rectangular slabs can be made more accurately by treating the entire panel as a structural unit than by analyzing individual strips as continuous beams.

(*b*) The use of an approximate method which assumes that the edge moments are distributed according to a sine curve, and which provides continuity at the middle of each edge, is feasible and gives maximum positive negative moments with sufficient accuracy for average loading conditions on any panel. Unusual loading conditions have not been considered.

(*c*) The equations that are used can be solved quickly by the method of iteration with the aid of an ordinary slide rule. Various loading conditions can be investigated by changing the constant term in each equation.

(*d*) The only apparent alternative to the foregoing approximate method appears to be in the development of tables of values for positive and negative moments for various panel arrangements and various ratios of live to dead load. Such a compilation of values by an exact method would be a long and laborious procedure; but it is possible with the aid of the curves and equations now available.

Acknowledgment

Most of the numerical data and diagrams in this paper have been taken from a dissertation by C. W. Pan on the "Analysis of Continuous Slabs," submitted in April, 1939, in partial fulfilment of the requirements for the Degree of Doctor of Science at the University of Michigan, Ann Arbor, Mich.

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PAPERS

THE SURVEYOR AND THE LAW

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Synopsis

It is not intended that this paper shall discuss, primarily, the law of boundaries. That is a very broad subject, capably treated in at least three books which, in whole or in part, are directly concerned with the subject.[2] The subject is also included, as incidental material to even broader subjects, in many treatises and compilations to which one commonly turns in efforts to learn the law. It is desired to present herein a discussion of those aspects of law which tend to indicate and to define the duties and the responsibilities of surveyors in their share of the work of establishing, describing, and recovering the boundaries of land. This is work in which there must be combined (with mutual understanding if best results are to be secured) the training, the experience, and the well-counseled efforts of practitioners of law and of engineering. This paper is a humble attempt to contribute a little to this end by discussion of some impacts of the law on the work of the engineer.

Introduction

Clearly, such a treatment as this cannot be exhaustive. Even though very limited in what it attempts, it is necessarily general in treatment. It is hoped that it will not be misleading, but it is too much to hope that none of its statements will be at variance from, or even contradictory to, that which is good law in some of the forty-nine different jurisdictions within the continental United States. It is assumed that, charitably and with good common sense, the reader will remember this fact; and also that, to be on safe ground, he must be ably advised on any given point concerning the law in the jurisdiction in which he is concerned. That the law of all other jurisdictions on that point might be otherwise would probably not then be material.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 15, 1941**.

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[2] "Law of Operations Preliminary to Construction in Engineering and Architecture" (especially Part III), by John C. Wait.
"The Legal Elements of Boundaries and Adjacent Properties," by R. H. Skelton.
"A Treatise on the Law of Surveying and Boundaries," by F. E. Clark.

Qualifications of a Surveyor

Let the reader consider a hypothetical surveyor, Mr. S, as he makes his preparation for his tasks, and go with him through some of the situations that he may be required to meet.

In most of the states of the Union there are now statutes providing that any person who is to do boundary surveying (or, as it is more commonly but less definitely called, "land surveying"), in the sense that he is to be in responsible charge of a boundary survey, must be registered or licensed by an examining board. These statutory requirements vary considerably from state to state, but they are easily discovered by referring to the code or the general laws of the state in question. Let it be assumed that if Mr. S is to practice in a state that has registration laws he has convinced the Board of Examiners that he is qualified to the extent that the statute and the Board require, and that he has been duly registered or licensed to practice.

Thus, the representatives of the state have in effect pronounced Mr. S to have at least the minimum personal and professional qualifications that should permit him to practice. What degree of care, skill, and judgment may a prospective client properly expect of him; and what may be the consequences if he fail to measure up in these respects to the requirements of some job that he may undertake? The courts answer this question by quoting the phrases that have long been used in the law to define that standard of conduct which separates negligence from due care: He must "exercise that degree of care which a skilled civil engineer of ordinary prudence would have exercised under similar circumstances."[3] The engineer will probably wish for a more specific, more tangible criterion that could be generally applied; but he will seek in vain. This standard must be used and interpreted in the light of the circumstances of each situation. In Ferrie vs. Sperry[3] the court says,

> "* * * it was important that the jury should know what such an ordinarily prudent engineer would do under the circumstances of this case. Might he simply examine the land records and muniments of title, and observe the fixed monuments and evidences of present occupancy and ownership; or was he bound to scour the neighborhood to learn whether there had been adverse occupancy and claims of ownership of any part of the premises covered by his client's deed by others than the latter's predecessors in title appearing of record? * * * The jury would not know, unless informed by evidence, and such evidence was admissible."

Thus must the question be resolved if it finally come to a court—by expert testimony of those who profess to know what should be done.

Duties and Liabilities of the Surveyor

The present interest, however, is in having Mr. S do whatever he should do, and in keeping him out of court. Mr. S, then, should make as thorough an examination of all reasonably available sources of information as circumstances seem to permit—and err a little on the side of safety, in diligence of search. This applies to matters of record, to evidences of boundary or of occupancy to be found on the ground, and to any other items of information

[3] Ferrie vs. Sperry, 85 Conn. 337, 82 A. 577.

that his eyes or his ears can discern. He must then pursue his field work and computations by such methods as will secure a precision appropriate to the job, and will eliminate the possibility of there remaining that bugbear of all engineers, a mistake. He should make very clear to his client the situation that he finds, in order that, if necessary, the client may use his own judgment in seeking legal advice as to the significance of conditions of occupancy, conflicting claims, and other matters that may override metes and bounds in fixing the lines.

True it is, the owner wants and expects to have the proper boundaries marked and reported to him, and does not want a list of conflicting items of evidence; and to the credit of the judgment of surveyors it may be said that that is what he usually gets. It is this "judicial function" of the surveyor (thus so aptly termed by Mr. Justice Cooley, of the Supreme Court of Michigan) that calls for experience and sound legal judgment, as well as engineering ability, on the part of a surveyor.

Suppose Mr. S fails to measure up to this required standard of care and skill. It has been generally held that he is answerable in damages to his client for the loss or injury that results from such failure—at least for any loss coming from a use of the land in a manner known to the surveyor to be contemplated at the time of the survey. A surveyor has been required to pay the cost of moving an apartment building, built to the line which he had marked, back some five feet to the true line.[4] The difficulty in that case was that of a mistake in measurement,—which is undoubtedly a clear failure to use due care. The surveyor in this case was not relieved of his burden (to a cost of some $1,267) because he had charged only $12 for the survey—a rather impressive commentary on the undesirability of cheap surveys, from the standpoint either of the surveyor or of the client. The custom of giving a certificate of accuracy, for which a higher fee was charged, was found "wholly immaterial" in this case.

It has been held that if a person is employed as a municipal official (for example, a city engineer) and does work for private persons as a function of his office, he is liable to them only in case of loss from his negligence or fraud; but that if a person works in the capacity of a private practitioner he is liable as well for loss resulting from want of skill.[5] Cases in other jurisdictions seem to indicate that the more general rule is that a surveyor undertakes to bring reasonable skill and knowledge to his work, be it done in either official or private capacity, and that he will be liable for loss resulting to his client because of his failure to do so.[6] The municipality, however, is not liable if its engineer does work erroneously at the request and for the benefit of a private person (in the case cited, setting stakes erroneously to show street grades in front of a house lot); even though he be bound, in his official capacity, to do such work upon request.[7]

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[4] Taft vs. Rutherford, 66 Wash. 256, 119 Pac. 740, 38 L.R.A. n.s. 1043.

[5] McCarty vs. Bauer, 3 Kan. 237.

[6] Highway Commissioners vs. Beebe, 55 Mich. 137, 20 N.W. 826.

[7] Waller vs. City of Dubuque, 69 Iowa 541, 29 N.W. 456.

It is believed that the duties and liabilities to his client of a surveyor who has been registered or licensed to practice do not differ from those of an unlicensed practitioner in a jurisdiction where license is not required. Of course, one who is licensed but is subsequently found incompetent may be prevented (by such process as is provided in the statutes of the state in question) from continuing to practice, or he may have his license revoked.

Surveyor's Right of Entry

Now, suppose that Mr. S, duly advised as to the care to be used, is proceeding with his survey. It may be practically necessary for him, either in actually running out the lines or for the purpose of reaching control monuments to which to refer his survey, to go on other land than that of his client. Surveyors regularly do this, but have they any right to do so?

If the surveyor is conducting in a reasonable manner a survey for an undertaking directed or authorized by act of the legislature, and if "the entry is reasonably necessary, and * * * is but a temporary one, and accompanied with no unnecessary damage," there is no trespass.[8] It is generally held that a body having the right of eminent domain in the furtherance of some project may, without payment of compensation, enter upon private land to make preliminary surveys therefor, and that such entry is not a trespass; and the statutes of some states so provide. Under such rulings, surveys for highways, for railroads, and for other improvements have been made; and the officials of New England towns have performed their duties of periodically "perambulating" their boundaries.[9] Of course, compensation must be made for any actual damage done, and any excessive or unnecessary damage will be enjoined.[10] In at least one case it has been held that where the state's officers (the state engineer and those working under his direction), in making a survey directed by the legislature of a disputed boundary line of certain counties, exceeded the amount of damage that was reasonably necessary, the state was not liable for the excess damage but the state's officers were.[11] In this case the damage complained of consisted in cutting through plaintiff's woods a "slash" $3\frac{1}{4}$ miles long and from 5 to 25 ft wide, for the purpose of establishing a base line. A referee in the case found that the method used was proper, and the best for the purpose, and that the work was skilfully and carefully done. The damage amounted to about $500. It should be mentioned that in this case both the referee and the lower court had found for the defendants (the surveyors) and that in the Appellate Division there was a strong dissenting opinion.

In the absence of any official mission or the possession of any official character, a surveyor who enters the land of another without permission is as much a trespasser as any other intruder would be. The action of trespass

[8] Winslow vs. Gifford, 6 Cush. (Mass.) 327.
Dill. Mun. Corp. 4th Ed. Sec. 614.
Edwards vs. Law, 71 N.Y.S. 1097, 63 App. Div. 451.

[9] Fox vs. Western P. R. Co., 31 Cal. 538.
Burrow vs. Terre Haute & L. R. Co., 107 Ind. 432, 8 N.E. 167.
Brigham vs. Edmands, 7 Gray (Mass.) 359.

[10] Dancy vs. Alabama Power Co., 198 Ala. 504, 73 So. 901.

[11] Litchfield vs. Bond, 186 N.Y. 66, 78 N.E. 719.

will lie even though no actual damage be done to the land or to any of its appurtenances.[12] Perhaps it may be apt to remark that, again, it is much to the credit of the ability of surveyors to "get along" amicably with those on whose land they almost necessarily trespass, that there are in the books very few cases of actions of trespass against surveyors, or because of the mere act of surveying.

This is the situation under the common law, but in some states legislative enactments have brought a modification. As an example, Section 2586 of the Public Laws of Vermont (1933) is quoted:

> "In cases wherein the title to lands, tenements or hereditaments may come in question, the practical surveyor with the necessary assistants, employed by any of the parties to such disputed title, may enter upon such lands or real estate, or other lands, for the purpose of running doubtful or disputed lines and locating or searching for monuments and ascertaining and deciding the lines and monuments of a survey, doing as little damage as possible to the owners of such lands."

Laws are now appearing on the statute books of a few states giving to engineers or surveyors the right to enter upon private land, whereon is a control survey station, for the purpose of using the station for engineering or surveying purposes. The laws usually require that no unnecessary damage be done, and that any anticipated damage be paid for in advance.[13]

A conclusion, drawn from all the foregoing and from the common experiences of surveyors, for the guidance of Mr. S, may well be summarized somewhat as follows:

The survey of a boundary line will almost inevitably require going on other lands than those of the client. Ordinarily, if care is used to cause no appreciable damage, no objection will be raised by the owner or occupant of such land; and if and when such objection is raised a frank explanation of the surveyor's need of permission to enter, a sincere promise of due care, and an explanation of the disinterested and non-partisan character of his work and conclusions will often secure the needed permission. Such entry, however, cannot properly be forced unless the surveyor is acting in an official capacity or for a legally-authorized undertaking; or unless it is under such a statute as that just mentioned. The invasion should be as brief and unobjectionable as practicable, limited strictly to the purposes of the survey. No reasonably avoidable damage should be done, and any unavoidable damage should be explained and compensation made.

Finality of a Survey

As Mr. S proceeds with his survey he may reach very firm and, in his opinion at least, very well-founded opinions as to where the lines with which he is concerned are, or should be. It is usually expected by his client that such will be the case, and that monuments of some kind will be set to represent

[12] Dougherty vs. Stepp, 18 No. Car. (1 Dev. & Bat. Law) 371.

[13] An excellent example of such a law is included in Chapter 628 of Laws of Maryland 1939. The part of that chapter to which reference is here made will become Sections 32 to 35, inclusive, of Article 91 of the Annotated Code of Maryland (1924 Ed.).

such conclusions. However, Mr. S may find that his conclusions disagree from those of some other survey or from evidences of occupancy and use. May he, if he is "sure" he is right, "establish" the boundary? The same question is sometimes put differently by a client: "Is your survey 'legal'? Will it 'hold'?" The answer probably should be that no survey, unless it be of an original and official character such as were the original surveys of the United States Public Lands, is "legal" in the sense in which people use that term in this connection; nor will it necessarily "hold" without the occurrence of one of two things: (1) The acceptance of the result by those whose interests are affected, or (2) a decree of court that the survey represents the proper solution of the boundary problem involved.

No modification of this statement is to be made because the surveyor may possess some official character, such as that of a city or county surveyor, nor because that which such a surveyor has concluded is sanctioned by his superiors (the county commissioners, for example). He has no more authority to conclude a matter of boundary than has a private surveyor,—or a blacksmith. Property rights must be settled by those who own them or by the courts. The Michigan court has well said,[14]

> "It is quite manifest that in this case there seems to have been an idea that the ex parte action of surveyors and commissioners was entitled to credence and authority upon the true lines and the fact of encroachment [of a fence and buildings alleged to be within a highway]. Neither of them can affect vested rights or settle controversies. They may be useful witnesses, when they speak of matters with which they are familiar, but they have no greater right than anyone else to determine starting points or boundaries. * * * The rash and vexatious acts of local officers not under the process of law are fruitful sources of bad blood and litigation."

(Since the boundary of a highway has been mentioned let the caution be noted that cases in which boundaries of highways are concerned are ordinarily excluded from the effect of the doctrine of acquiescence or that of adverse possession.[15])

Now that the legal situation has been outlined, let the statement be made —once more to the credit of surveyors in general—that, as a practical matter, many a boundary dispute has been settled by surveyors, many boundaries have been "established" by them; and may such continue to be done. In honest, competent hands, this is one of the most practical, most useful results of the exercise by the surveyor of his unofficial "judicial function."

Arbitration

Now let it be supposed that, at the end of his work over some boundary, Mr. S does reach a conclusion which differs from that of some one else, or to which not all items of evidence—or "calls"—agree; or perhaps he is forced to the conclusion that it would be disproportionately expensive, or even practically impossible, to execute surveys that would enable any honest surveyor to say, "There are the stakes which with reasonable certainty and precision mark the

[14] Gregory vs. Knight, 50 Mich. 61.
[15] Langle vs. Branch, 193 Iowa 140, 185 N.W. 28. Subject is discussed in 6 A.L.R. 1210.

line." It may often be true, in such a situation, that it will be possible to say, with relatively small limits of variation, approximately where the line is. Evidences of occupancy might accomplish that, or the differing conclusion from another survey may not represent a large difference. Mr. S's best service, in the interest not only of his client but of all who are concerned, may now lead him to suggest that the best results, both from the standpoint of property rights and also with reference to the amicable relations of all concerned, will be reached by compromise or by arbitration. If interested persons will either come to an understanding of all the conflicting elements involved and then hit upon and accept a solution, or if they will let the surveyor propose a compromise solution, with the reasons therefor, and then accept it, the Gordian knot will be cut,—and the chances are good that all will be happier than at the reaching of a conclusion in a court of law; for, after all, the decision of a court would have to be based on evidence, and it is unlikely that the court would have any more evidence, or have it presented with less bias, than that now in the possession of the surveyor. No suggestion is here made that any substantial, well-founded rights of a client, properly enforceable in a court, should be recommended for surrender; it is believed that the proper setting for the procedure recommended has been made clear. Obviously, to function acceptably in the rôle suggested, Mr. S can use all the legal and engineering training and experience and judgment at his command. To carry the matter to its best conclusion, as a competent and fully-informed court would do, will be another instance of his exercise of his judicial function.

What must be done to make such an acceptance of a compromise solution binding? How should it be perpetuated?

With reference to the first of these two points, it seems clear that nothing further than the parol—oral—agreement of the owners concerned is needed to conclude the matter and to make their agreement binding upon them and upon their subsequent grantees. This is not a situation in which, under the statute of frauds, a writing is necessary to make the agreement effective. In a brief, but clearly-expressed, case[16] the Supreme Court of Arkansas has said,

> "This court has held in accord with the weight of authority that, where there is doubt or uncertainty or a dispute has arisen as to the true location of a boundary line, the owners of the adjoining lands may, by parol agreement, fix a line that will be binding upon them, although their possession under such agreement may not continue for the full statutory period."

The Supreme Court of the United States has held[17] that such an agreement is "not a contract for the sale or conveyance of lands. It has no ingredient of such a contract." Adjoining owners hold up to a boundary thus fixed by consent by virtue of their title deeds, not by virtue of a parol transfer between themselves.[16,18]

[16] Robinson vs. Gaylord, 182 Ark. 624, 33 S.W. 2d. 710.
[17] Boyd vs. Graves, 4 Wheat. 513, 4 L.Ed. 628.
[18] Cutler vs. Callison, 72 Ill. 113.

Such an agreement to fix a boundary line is void if the owners know or if one of them knows that the agreed line is not the true line.[19] It is then in effect a transfer of land, or perhaps even a fraud on the part of the one who knows. However, that the line could be ascertained by an expensive survey, the cost of which the owners thus avoid, does not prevent the uncertainty that must exist to make the agreement valid—and valid "even though the true line should afterward be ascertained."[20,21]

It may be that the parties will prefer, rather than to try to agree upon a location themselves, to submit the question of boundary to arbitration. Still assuming that the arbitrator is an honest and a competent surveyor, that may be the very best solution. That it is a valid one is amply supported by decisions.[22] It is not necessary that the agreement to do so be in writing[23]; although, to prevent misunderstanding, a writing may sometimes be desirable. When a writing is used, it should be carefully drawn, and it must be taken to be the entire basis of the submission to arbitration. No parol modification would be effective, under the statute of frauds,[24] or the parol evidence rule.

The award, resulting from an arbitration conducted in accordance with the terms of the submission, is valid and conclusive.[25] It cannot be impeached because of an honest mistake of judgment on the part of the arbitrator.[26]

This fact, and the aforementioned fact that a location agreed upon will stand even though subsequent developments show that it was not the true line, are only corollaries of the elementary and fundamental principle of boundary law that a boundary once authoritatively established and adopted remains in that place, regardless of mistakes of measurement or of judgment made in arriving at that location. This principle is too thoroughly established and too well known to require citation of authority or discussion here. It is comparable in importance to, and is directly connected with, the fundamental rule for interpretation or construction of land descriptions and for search for land boundaries—namely, that which is to be accomplished if possible is to ascertain and to carry into effect the intent of the parties. To effectuate these fundamental principles, all rules and all "order of calls" are enunciated, and to them all such rules must yield.

All that is absolutely required to be done to make a location by agreement or by arbitration effective,—to perpetuate it,—is to describe the boundary with certainty.[27] However, as a surveyor, Mr. S should know that to provide a

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[19] Moran vs. Choate, 69 S.W. 2d. 994, 253 Ky. 470.
Johnson vs. Buck, 46 P. 2d. 771, 7 Cal. App. 2d. 197.

[20] Lynch vs. Egan, 67 Neb. 541, 93 N.W. 775, 777.
Sobol vs. Gulinson, 94 Colo. 1, 28 P. 2d. 810.
Blanford vs. Biven, 254 P. 1030, 123 Kan. 269.
Ferrill vs. Bryson, 37 S.W. 2d. 841 (Tex. Civ. App.).

[21] The matter is treated, and many citations are given, in 69 A.L.R. 1430, 1433, and in 11 C.J.S. Boundaries Sec. 67.

[22] Turner vs. Spicer, 249 S.W. 1038, 198 Ky. 739.
Robbs vs. Woolfolk, Tex. Civ. App., 224 S.W. 232.

[23] Hill vs. Walker, Tex. Civ. App., 140 S.W. 1159.
Smith vs. Seitz, 89 A. 257, 87 Conn. 678.

[24] Tabor vs. Craft, 116 So. 132, 217 Ala. 276.

[25] Turner vs. Spicer, 249 S.W. 1038, 198 Ky. 739.

[26] Rottman vs. Taft, 204, N.W. 585, 187 Wis. 558.

[27] Giddings vs. Hadaway, 28 Vt. 342.

permanent remedy, to prevent a recurrence of the trouble, the line should be adequately and "permanently" monumented and platted, and the plat should be recorded with written, signed, and acknowledged acceptance of that which is shown thereon, or accompanied by a written submission to arbitration and a certificate by the arbitrator that the plat shows the result of the award.

Now, lest Mr. S might (from the emphasis thus far laid on agreement or acquiescence) think that the author is saying that such acquiescence concerning another boundary than the original surveyed line destroys all significance of the latter, two points should be kept in mind:

The first is merely a matter of interest to be noted in passing; namely, it has been held[28] that the conveyance of land by government subdivisions, without any representation of quantity, conveys only that portion included within boundaries acquiesced in for more than the period of the statute of limitations, and visibly marked, even though it is less than the full area of such subdivisions.

The second point, of more general consequence, is that although such acquiescence or agreement in a boundary line is binding on the parties thereto and on those claiming under them[29] (grantees, heirs, etc.) only such persons are bound by the acquiescence or agreement.[30]

This conclusion has some implications that would lead out of the immediate field of the present discussion; but the point to be noted here is that, although agreement, acquiescence, or adverse possession may have taken away from the original location of a section line, for example, its character as a boundary line between the contiguous parcels, it may still be essential to locate the original position of the section line and of the section corners thereon as sources from which to work in locating other lines or corners. Even though the significance of the first lines as immediate boundaries may have disappeared, they are just as effective (if they can be found) as they ever were in every other respect.

The Surveyor as an Expert Witness

It is to be hoped that Mr. S can conclude for his client the questions of boundary with which he may be concerned without the necessity of the latter becoming involved in an action in court. There are some situations, however, in which only a court can settle the matter properly. In such a case Mr. S should be able to be of material assistance to his client's attorney; and this he can do better if he has some knowledge of the law concerned. Attorneys, especially those in general practice, are not always entirely familiar with matters of boundaries, with the engineering terms involved, with the practicability or the difficulty of accomplishing, in special situations, things which seem theoretically possible. It should not disturb the professional pride of a lawyer to review his trial brief with the surveyor; and the writer has known of cases

[28] Johnson vs. Trump, 161 Iowa 512, 143 N.W. 510.

[29] Amber vs. Cain, 110 N.W. 1053 (Iowa, 1907).
Rydalch vs. Anderson, 37 Utah 99, 107 P. 25.

[30] Ivey vs. Cowart, 124 Ga. 159, 52 S.E. 436, 110 Am. St. R. 160.
Sawyer vs. Coolidge, 34 Vt. 303.
Kyte vs. Chessmore, 94 Kan. 611, 146 P. 1152.

in which, as he believes, the procedure and possibly the result could have been improved had that been done. Ordinarily the surveyor will be used as an expert witness. That this may be done effectively requires that the examining attorney and the prospective witness go carefully over the matter beforehand. The witness may well explain, first (if it is not already known to the attorney), just what the witness' testimony should show. Questions can then be framed which will clearly and expeditiously bring these facts before the court. If the surveyor listens carefully during the trial, especially as other surveyors give their testimony, he can frequently be of material help in catching points or suggesting questions of a technical nature. None of this savors in any way of shady practice or of any variation from the truth. No reputable attorney will ask or expect a reputable engineer to become involved in any "sharp" practice to gain a technical advantage, nor to vary from the truth as he sees it in giving his testimony.

It may be worth while to inject here the statement that even in states which have statutes providing that no surveys shall be legal evidence unless made by a county surveyor or his deputy, or by the United States or by the state, it has been held that such statutes do not preclude the admissibility of testimony of a competent private surveyor concerning a survey made by himself, or concerning a plat of it that he has made.[31]

It is clear that, in giving testimony, a surveyor may refresh his memory by referring to his field notes or to a plat made from his notes. This is true even if the survey or the plat were not made in all respects in accordance with a statute providing methods therefor.[32]

Admissibility of Plats and Field Notes

The following excerpt from Corpus Juris Secundum[33] states concisely the general situation concerning the admissibility as evidence of maps and plats. The citations here given are only a few of those available:

> "As a general rule, maps or plats of surveys are admissible in evidence.[34] These include maps or plats made by the county surveyor or other official surveyors[35]; plats of unofficial surveys when made by a qualified surveyor[36]; maps from the land office,[37] although made subsequent to the beginning of the action[38]; and maps made by order of the state land board. Likewise, plats and maps attached to and made a part of deeds or grants or referred to therein, either expressly or by clear implication,[39] maps which have been recognized as correct by a former owner of the land,[40] and ancient

[31] Von Eime vs. Fuchs, 8 S.W. 2d 824, 320 Mo. 746.
Cody vs. Black, 192 P. 282, 97 Ore. 343.

[32] Krider vs. Milner, 99 Mo. 145.
Gee vs. Sherman, 221 Mo. App. 121, 126.

[33] 11 C.J.S., Boundaries Sec. 113, p. 709.

[34] Martin vs. Hughes (Pa.) 98 Fed. 556, 39 C.C.A. 160.

[35] Price vs. DeReyes, 119 P. 893, 161 Cal. 484.

[36] Heinrichs vs. Terrell, 21 N.W. 171, 65 Iowa 25.
Nickel vs. Chapman, 158 N.W. 90, 163 Wis. 348.

[37] Giraud vs. Huffman, Tex. Civ. App., 46 S.W. 2d 367.

[38] Thatcher vs. Matthews, Tex. Civ. App., 183 S.W. 810.

[39] Daniel vs. Finn et al., 119 S.E. 307, 156 Ga. 310.
McDaniel vs. Leuer, 230 S.W. 633.

[40] Nichols vs. Turney, 15 Conn. 101.
Gates vs. McCormick, 97 S.E. 626, 176 N.C. 640.

maps and plats when duly authenticated,[41] are admissible to prove a disputed boundary. A map or plat referred to in the testimony is admissible as explanatory of it.

"In all cases the map or plat must be verified or authenticated,[42] and in the case of a surveyor's plat proof offered that the surveyor making the same actually traced the lines on the ground as depicted."[43]

However, in one case[44] it was held that

"* * * plats and surveys attached to certain deeds and made by reference a part thereof were properly admitted in evidence with the deeds, notwithstanding the attached documents were not proved by the surveyor nor their correctness otherwise established. * * * Aliter [It would be otherwise] if the plats and surveys had not been made a part of the deeds themselves."

With reference to the plats or field notes of surveyors now deceased, one court[45] has held that:

"* * * ancient plans of lots, showing the original lotting and boundaries assigned, also ancient minutes of surveyors, which were made by capable independent surveyors, having no interest to misrepresent, and which have been preserved, and are produced from reliable sources, may likewise be used on trial, when the boundaries of lots to which said surveys refer come in question."

To the same effect are cases from other jurisdictions.[46]

Some states have by statute made the matter relatively clear. An example is the following section from the Iowa Code:

"11295. *Field notes and plats.* A copy of the field notes of any surveyor, or a plat made by him and certified under oath as correct, may be received as evidence to show the shape or dimensions of a tract of land, or any other fact the ascertainment of which requires the exercise of scientific skill or calculation only."

In situations quite analogous to those affecting the introduction in evidence of plats, the field notes of an authorized survey, when duly authenticated, are at common law admissible.[47] Also, at common law, the field notes of a deceased surveyor are admissible[48]; but at least one court has held[49] that a statutory procedure, recently enacted, under which county surveyors' field notes may be admitted, abrogates the common law which admitted the notes of surveyors, either public or private, and that therefore all other field notes than those of county surveyors are excluded. This would seem to be an undesirable result.

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[41] Brown vs. Metcalf, 215 Mass. 289, 102 N.E. 413.
Donohue vs. Whitney, 133 N.Y. 178, 30 N.E. 848.
Burgan vs. Siegman, 9 Ohio App. 84.

[42] Kearce vs. Maloy, 142 S.E. 271, 166 Ga. 89.
Cashion vs. Meredith, 64 S.W. 2d 670, 333 Mo. 970.

[43] Marcone vs. Dowell, 173 P. 465, 178 Cal. 396.
Maxcy vs. Norsworthy, Tex. Civ. App., 19 S.W. 2d 926.

[44] Hill vs. Snellings, 154 S.E. 156, 41 Ga. App. 585.

[45] Smith vs. Forrest, 49 N.H. 230, 239.

[46] Hamilton vs. Smith, 74 Conn. 374, 50 Atl. 884.
Prouty vs. Tilden, 164 Ill. 163, 45 N.E. 445.

[47] Giraud vs. Huffman, Tex. Civ. App., 46 S.W. 2d 367.
Neill vs. Ward, 153 A. 219, 103 Vt. 117.

[48] Stewart vs. Stephenson, 89 S.E. 1060, 172 N.C. 81.

[49] Peck vs. Molhoek, 228 N.W. 721, 249 Mich. 360.

The older practice has been to take field notes in a bound book, and it is undoubtedly that form of field book which has usually been introduced in evidence. Mr. S may prefer a loose-leaf field book, and a question may arise in his mind as to the admissibility of notes taken on such loose sheets. Whatever may be one's opinion of the judgment of Mr. S if he decides to use loose sheets, even in some form of "binder," for field notes, it is still probable that if the sheets are properly identified and authenticated they would be admissible under the same circumstances that would admit a bound book. The writer knows of no case directly in point. The practical difficulty of authenticating and handling half a dozen separate sheets bearing the notes of a survey, as compared with the corresponding use of a book, would seem likely to produce some annoyances and danger of loss. The foregoing opinion concerning admissibility is based, by analogy, on modern usage concerning office records. By statute in some states, and by court decision in others, loose leaves[50] or cards[51] bearing office records are admissible as evidence when properly identified.

It is true that loose, miscellaneous memoranda have generally been excluded, but "the admissibility or inadmissibility of such entries would appear to rest more on their form than on the nature of the book in which they are made."[52]

Definition and Ownership of Field Notes

It has seemed that not all usages are quite consistent as to what is meant when the term "field notes" is used. Field notes are defined in Corpus Juris as "Notes made by the surveyor in the field while making the survey, describing by course and distance, and by natural or artificial marks found or made by him where he ran the lines and made the corners."[53] "Words and Phrases" uses the same definition, referring to the same case therefor; and with this definition a surveyor should have no quarrel. However, in spite of this plain statement in this case, apparently taken by both authorities quoted as the best available definition of field notes, some usages seem to refer to a description subsequently "written up" from the notes which were actually put down at the time measurements were made or objects were observed. Some substance is lent to this view by an inspection of the books said to contain the "original field notes" of original government surveys of the public lands. To make a very conservative statement, these books, and the neatly ink-written narratives that they contain, do not seem likely to have been with the surveyors in the field under some of the conditions of weather and work described therein. The terms "field book"[54] and "field notes" are sometimes applied to books and contents that are obviously office (usually a public office) records.

These observations are made in reference to suggestions sometimes heard that the field notes of a survey should be turned over to the client upon com-

[50] Queen City Sav. Bank vs. Rayburn, 163 Fed. 597 (aff. 171 Fed. 609, 96 C.C.A. 373).
Wylie vs. Bushnell, 277 Ill. 484, 115 N.E. 618.
United Groc. Co. vs. Dannelly, 93 S.C. 580, 77 S.E. 706, Ann. Cas. 1914D, 489.

[51] Haley, etc. Co. vs. Del Vecchio, 36 S.D. 64, 153 N.W. 898, L.R.A. 1916B, 631.

[52] 22 C.J. 871, Sec. 1046.

[53] State vs. Palacios, Tex. Civ. App., 150 S.W. 229, 236.

[54] Neill vs. Ward, 153 A. 219, 103 Vt. 117.

pletion of the work as part of that for which he presumably has paid. It is the opinion of the writer, and of others with whom the matter has been discussed, that this would not be the best arrangement, either from the viewpoint of the surveyor or from that of his client. It is the opinion of the writer that Mr. S should turn over to his client, in the most useful form practicable, all the essential information that has been accumulated at the expense of the client. The most useful form (in addition to whatever physical evidences of the work are left on the site) will probably be a plat. A written description of the boundary may sometimes be useful, and to this some persons might give the name "field notes." A more probable label would be a "survey," which name seems to be used indiscriminately to refer either to the work done or to the report of such work in the form of a description.

This plat or other report should be as complete as circumstances will permit. It should be sufficient to enable any qualified person to identify the land or to recover an obliterated bound at any time in the future. The withholding of any essential data for the purpose of insuring reemployment on any future work in the vicinity cannot be too strongly condemned; but the field book—the book used in the field at the time the work is done, and in which the data are set down, usually in pencil, as they are observed—may better stay in the hands of Mr. S. It is quite probable that these notes will be in a tabulated form, in which they would be of little use to a client who was not himself familiar with survey work and notes. They probably contain data concerning random traverses, and the like, which would only be confusing to him. Moreover, cases which have been cited herein indicate, incidentally, that field notebooks are frequently found in the possession or among the effects of those who took the notes. The writer has neither read nor observed in his own practice anything that would lead him to believe that there has been, or is, any custom of turning field notes of private surveyors over to their clients; nor has he seen anything in statute or reported decision to cause him to think that it is expected—let alone required.

Of course, the terms or circumstances of the engagement of an engineer, either as a public official, such as a county surveyor, city engineer, etc., or as an employee in a private office, often make it clear that the notes he takes are not his private property but belong to the office. The remarks that have been made concern the engagement of a private surveyor or engineering firm on private work for a client.

Surveyor's Report to His Client

One of the final matters with which Mr. S will be concerned in a survey is the furnishing of an adequate description of the land surveyed as one of the chief elements of his report to his client. The two purposes which this description must serve should be constantly kept in mind:

(1) It must make convenient and certain the identification of the tract of land for the purpose of tracing title; and

(2) It must be sufficient to enable any competent engineer to restore conveniently at any future time a bound (or all the bounds) which may have been obliterated.

For the first purpose, in order that it may be incorporated in a deed, a verbal description, giving general location, and perhaps adjoiners, is essential. It should be as brief as practicable, should be such as may conveniently be used over and over in successive instruments, each of which should refer to a preceding one; and, as an item of paramount importance, it should incorporate a plat by referring to it so as to make the plat a part of the deed description. That the complete description given on the plat will then determine the boundaries of the land conveyed is now too well settled to require authority for the statement here.[55] For the most part the cases cited in discussions of this matter are situations in which some special point had caused the raising of a question; for example, it is immaterial by whom the map was made[56]; or the plat was not acknowledged or recorded[57]; or it was invalid.[58]

Needed for the first purpose mentioned, but absolutely essential in many cases for the second, is the incorporated plat. It is a matter of common knowledge and observation that graphical descriptions are often not only much more convenient than verbal ones, but much less likely to be misunderstood. To this rule land descriptions are no exception.

Mr. S has been assumed to be a competent engineer; therefore, one need not here review the specifications for a satisfactory plat,—other than the fundamental requirement expressed under Purpose (2). It should be so numbered or titled as to be referred to conveniently in the verbal part of the description; and it should be certified or acknowledged by the surveyor, and of course made a part of the record.

Surveyor's Final Obligation

Finally, with reference to the service which Mr. S should render his client, it may be stated that he should do his work so that it is unlikely that that particular job will ever have to be done again; and, paradoxically, it should be so done that it could be retraced conveniently by any competent engineer —stranger to the locality though he may be—at any time in the future. The first of these two requirements can best be attained by the use of distinctive and enduring monuments. To economize by setting wood stakes or other cheap monuments to perpetuate a survey is indeed being "penny wise and pound foolish." From a legal, engineering, or just good common sense standpoint, the best advice that can be given to a client in this respect (and all too often the client must be "shown") is to add to the cost of the survey enough to monument it well.

Concerning the second requirement—that of adequate description—beyond that which has been said, it is desired to emphasize but one point: It is sometimes true that plats which are complete in themselves are only inadequately located with respect to adjoining land. Until very recently no one satisfactory means of stating or showing such relative locations has been generally available. Now, with state plane coordinate systems devised for every state and permanently connected with the national control system of triangulation, such a

[55] See 9 C.J. 180, sec. 50.

[56] Finelite vs. Sinnott, 125 N.Y. 683, 25 N.E. 1089.

[57] Johnstone vs. Scott, 11 Mich. 232.

[58] Young vs. Cosgrove, 49 N.W. 1040, 83 Iowa 682.

means is at hand. As time and surveys go on, this means is becoming accessible to more and more land. Wherever this control is available, practically and economically, it should be used. Whenever practicable the plat should show in terms of coordinates in the state system the positions of one or more monuments on the boundary, and of one or more monuments on a control survey. Directions should then be stated in azimuths, referred, of course, to the same system. A statement on the plat should make it clear that this has been done. Statutes have been enacted in some states making definite the fact that such is an adequate description; but it is believed that, in the absence of any statute, the common law which would be applied in any state is clear that such a description is good. There are many decisions to the effect that a "deed must contain such a description * * * as will enable the property to be readily located by reference to the description"[59]; and also to the effect that "if the description of the land conveyed in a deed is such that a surveyor, by applying the rules of surveying, can locate the same, such description is sufficient * * *."[60]

It is for the quality of description specified by these quotations that the writer pleads: Such a description as "a surveyor"—any surveyor—can effectively use "by applying the rules of surveying"; not something that merely furnishes an opportunity for the exercise of ingenuity in the solving of a puzzle. The proper use of the state plane coordinate systems, now officially approved for this purpose by both the Society and the American Bar Association, is, in the opinion of the writer, the best means thus far made available for assuring the certainty and the permanence that are required.

[59] Saterstrom vs. Glick Bros. * * * Co., 5 P. 2d 21, 118 Cal. App. 379.

[60] Neves vs. Flannery, 149 So. 618, 111 Fla. 608.
Brooks vs. Pryor, 189 So. 675.

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PAPERS

COMPACTION OF COHESIONLESS FOUNDATION SOILS BY EXPLOSIVES

By A. K. B. Lyman,[1] M. Am. Soc. C. E.

Synopsis

The method of compacting loose, cohesionless foundation soils in their natural state by detonating buried charges of explosives has been used successfully at Franklin Falls Dam, Denison Dam, and Almond Dam. The use of this method, as reported herein, is intended to draw attention to a new economical and efficient method of obtaining a satisfactory degree of compaction in cohesionless foundation materials. To employ this method successfully, the materials must be in a condition approaching complete saturation, and under such conditions, this method is widely applicable to all loose, cohesionless structural foundations. The results obtained in investigations to date (1941) indicate that in addition to increasing the degree of compaction of the materials, the horizontal permeability of stratified deposits is greatly reduced. Further investigations and tests are needed to determine the limits of compaction for specific materials and whether optimum results can be obtained in fully or partly saturated material.

Introduction

Franklin Falls Dam on the Pemigewasset River near Franklin, N. H., was designed for the control of floods on the Merrimack River. In their natural state at this site, there are deposits of loose, fine alluvial sand, especially in the river terraces of the foundation area of the embankment that constitutes a part of the project development. Extensive investigations disclosed that the stability of the embankment would be considerably increased if these loose sand deposits could be consolidated to reduce the possibility of their failure by liquefaction due to disturbance caused by earthquakes, demolitions, or blasting in the vicinity.[2] Estimates of cost were prepared for various compaction

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by September 15, 1941.

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[2] *Civil Engineering*, April, 1940, p. 205.

methods, including that of driving piles and the use of large vibrators and tamping equipment to reduce the voids ratios of the loose terrace materials below critical values, as defined by Arthur Casagrande, Assoc. M. Am. Soc. C. E. Consideration was also given to the excavation of the loose materials and replacement by the same or other materials in a dense state by accepted compaction methods. The uniformly high cost of the treatments considered led the writer to investigate the feasibility of compacting the loose deposits by use of buried charges of explosives.

As a necessary prerequisite to the application of this method to compaction of loose soils it is necessary to consider: (*a*) Type of soil; (*b*) condition or degree of saturation of soil; and (*c*) depth of deposit. Although compaction by blasting was initially developed for the treatment of loose deposits of uniformly graded, slightly silty, fine to medium sands at Franklin Falls, subsequently sands containing a substantial percentage of silt have been successfully compacted at one other locality.

Saturation of the soil to be compacted, at least in the region of the explosive charges, is regarded as conducive to the best results. However, fairly effective compaction has been obtained in tests at the Denison (Tex.) Dam under conditions of partial saturation. The pore water probably acts initially as a tamping medium, thus facilitating the progression of the vibratory waves or shocks emanating from the charges. The explosion of buried charges induces a "liquefaction" of the soil mass, followed by an escape to the ground surface of excess pore water which acts as a lubricant to facilitate a rearrangement, in a more compact state, of the soil grains in the vicinity of, and overlying, the charge. Extensive explorations after blasting disclosed the absence of cavities which may have been created momentarily by blasting in the soil mass.

There are no apparent limitations to the depths of deposits that may be consolidated by means of explosives. On the contrary, deposits of sand at considerable depth may be compacted in this fashion, whereas they may not be readily accessible to other forms of treatment.

It is thought that where loose strata of sand greater than 30 ft thick are to be compacted, two or more tiers of small charges are to be preferred to one tier of large charges. For deposits less than 30 ft thick charges at depths, below the top surface of the mass to be compacted, of approximately two thirds of the thickness of the loose soil mass, will generally suffice.

The size and distribution of charges should be such as to shatter the soil mass thoroughly but not to the extent of creating permanent surface craters. A surface crust of impervious material inhibits the emission of the surplus pore water from the ground and must be disturbed, fractured, or removed. Following the dull thudding sound of the blast there is an upheaval of the ground surface in the range of influence, a fracturing of the surface, and the escape of gas and water for periods of from minutes to hours from crater-like openings similar to sand boils which appear on the surface. The primary surface settlement immediately follows the initial upheaval, and continued settlement is apparent for a few minutes. No doubt settlement continues for some time as part of the adjustment of the soil mass accompanying the continuing expulsion of surplus pore water.

The lateral distribution of charges should be based on results obtained from a series of single shots. A horizontal grid spacing of charges, 20 ft on centers, for the initial coverage, with an orderly interspersion of four subsequent coverages in the space intervals between prior coverages, proved satisfactory at Franklin Falls.

The degree of compaction obtained by blasting can be demonstrated by tests of undisturbed samples taken before and after the compaction operation. Undisturbed sampling below the ground-water table, although requiring expensive test pits, is necessary if the results are to be evaluated accurately. Observation of ground-surface settlement by means of stakes or anchored plates permits a generalized interpretation of the effectiveness of the blasting method. Moreover, such observations indicate the settlement for each blasting coverage and, hence, are a key to the number of coverages required.

The method of compacting loose, cohesionless foundation materials by means of buried explosive charges has been applied successfully to the river terraces which form a large part of the embankment foundation area at Franklin Falls Dam. Tests have also been conducted at Denison Dam in Texas and at the site of Almond Dam in western New York State.

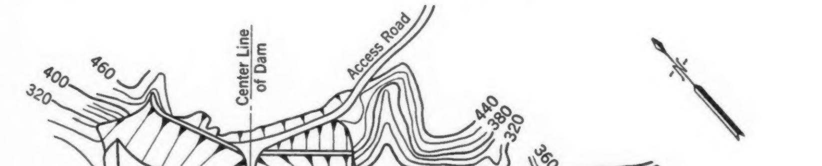

FIG. 1.—LOCATION PLAN, FRANKLIN FALLS DAM ON THE PEMIGEWASSET RIVER, FRANKLIN, N. H.

TESTS AT FRANKLIN FALLS DAM

The Franklin Falls embankment (see Fig. 1) is of rolled earth design, about 1,700 ft long and 130 ft high (maximum), with a height of approximately 110 ft above the terraces.

The embankment foundation area consists in part of two overbank flood plains or terraces, each about 400 ft wide, of loose, fine-to-medium, alluvially-deposited sands extending to depths of from 15 to 30 ft. Below this stratum are stable coarse sands and gravels. As construction of the dam proceeds the loose, cohesionless materials will consolidate slightly, but not sufficiently to reduce the fine materials below their critical voids ratios for the loads to be imposed. Apprehension was felt that these deposits might suddenly "liquefy" and flow from beneath the embankment due to disturbance caused by earthquakes, slides, settlements, or vibrations. Fig. 2(*a*) indicates the range of

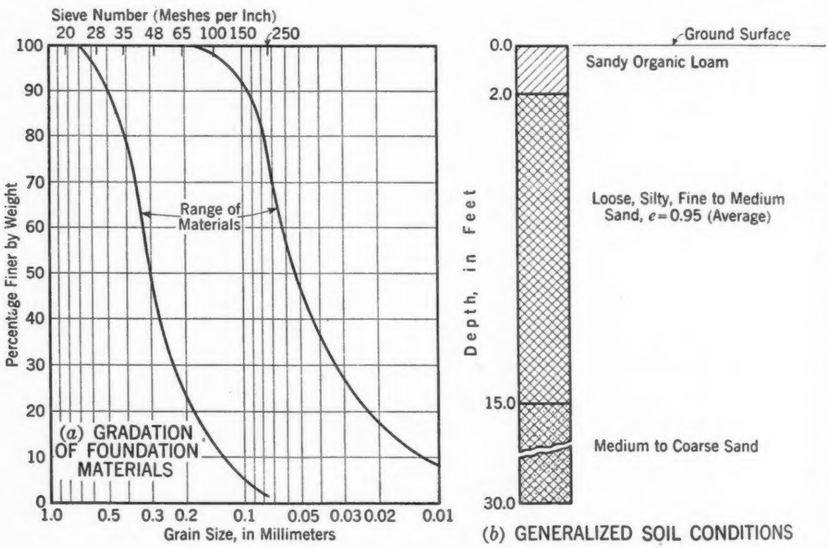

FIG. 2.—SOIL CHARACTERISTICS, FRANKLIN FALLS DAM

grading and Fig. 2(*b*) the distribution of the critical materials in the river terraces.

Alternate remedies to eliminate the hazard of foundation failure were dismissed from consideration because of their excessive cost until the possibility of compacting the loose terrace deposits by buried charges of explosives had been investigated. Preliminary test blasting consisted of the detonation of individual charges of dynamite ranging from 2 lb to 8 lb at depths of 10 to 16 ft below the ground surface; optimum consolidating effect was produced by charges of 8 lb of 60% dynamite at a depth of 15 ft below the ground surface.

A test area was next blasted with twenty-one charges at depths of 15 ft distributed into four coverages with final grid intervals of 10 ft. An average ground-surface settlement of the test area of 1.25 ft resulted. Tests of undisturbed soil samples taken from a test pit excavated to a depth of 8 ft in the center of the blasted area yielded the results indicated in Fig. 3. It is estimated that the average voids ratio before blasting was 1.0 and the degree of compaction 36%.

To confirm these results and further develop the method, a second test area, with five blasting coverages (see Fig. 4), resulted in an average ground-surface settlement of 2.0 ft, the compaction being effective over an estimated depth of 20 ft. Pits were excavated before (Pit 610) and after (Pit 611) blasting. Tests

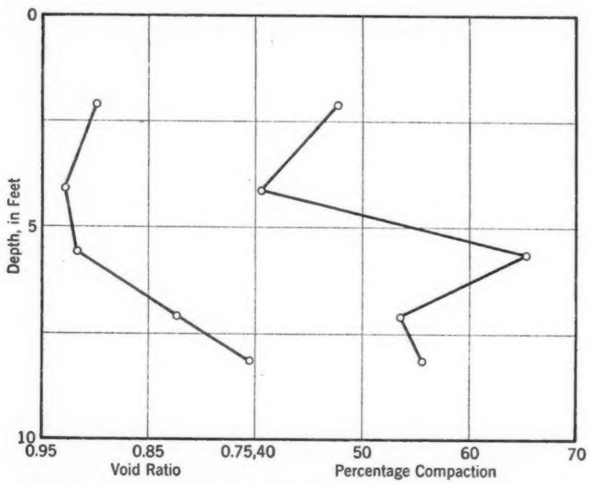

FIG. 3.—FIRST TEST AREA, TEST PIT 609; VOIDS RATIO VERSUS DEPTH

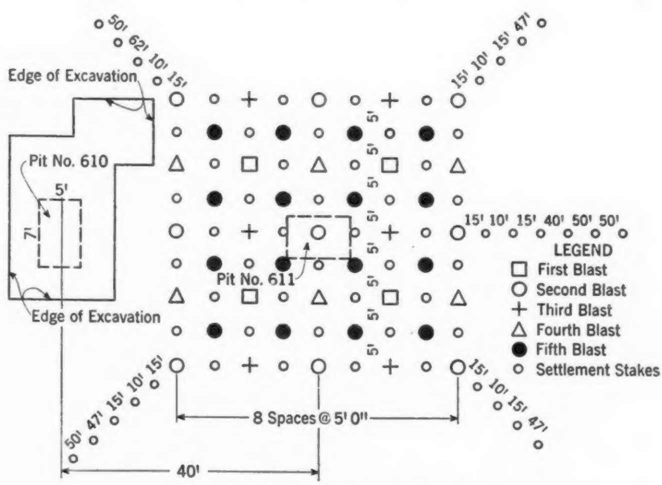

FIG. 4.—LAYOUT OF SECOND TEST AREA

of undisturbed soil samples indicate changes in voids ratio and degree of compaction as plotted in Fig. 5. No evidence of cavities was revealed by the test pits excavated after the blasting operations. A total of six soil samples taken at each elevation indicated a fairly uniform degree of compaction laterally at each depth.

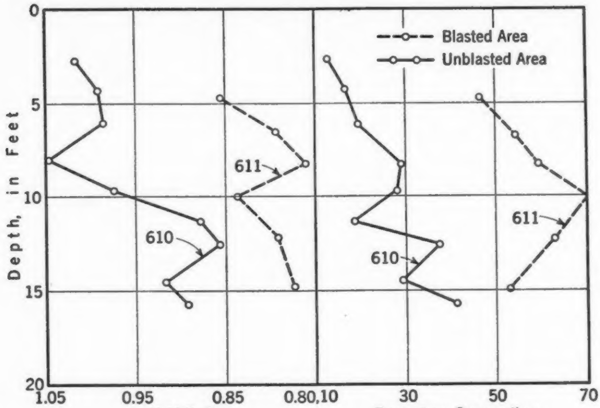

FIG. 5.—SECOND TEST AREA, TEST PITS 610 AND 611; VOIDS RATIO AND DEGREE OF COMPACTION VERSUS DEPTH

Permeability is reduced substantially by the compaction operations and is attributable to two causes: First, the reduction in void spaces between soil grains; and second, the breaking up of lensing and stratification of the natural deposits. The relative magnitude of these two effects is shown in Fig. 6. Field pumping tests before and after compaction operations from identical well

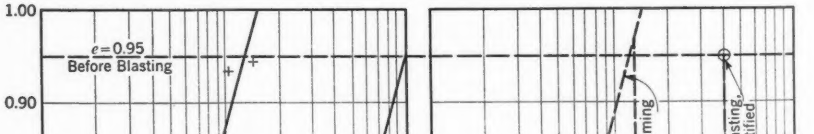

FIG. 6.—LABORATORY PERMEABILITY CURVES ON REMOLDED SAMPLES; FIELD PUMPING TEST

point systems around test pits 610 and 611 resulted in quantities of flow of 400 gal per min and 75 gal per min, respectively. Laboratory tests indicate that the reduction in voids ratio would account for only 45 gal per min, and the remainder of the reduction of 280 gal per min is attributed to the breaking up of lensing and stratification which indicates the importance of altering this characteristic of highly permeable soil deposits.

As a result of these tests it was concluded that the loose sand terraces could be compacted satisfactorily by blasting methods. Each of the terraces, measuring approximately 400 ft by 850 ft, was compacted by using a total of 12,000 explosive charges. On the west terrace the average ground-surface settlement was 2.0 ft in a depth of 20 ft of loose materials. On the east terrace where the loose materials extended to an average depth of 25 ft, the settlement was greater, averaging about 2.5 ft.

Excavation of trenches in the compacted terraces for the foundation drains beneath the embankment afforded opportunity for extensive undisturbed sampling. The fifty-seven samples tested indicate an average voids ratio of 0.780 for the compacted materials in the west terrace and 0.736 in the east terrace, with respective degrees of compaction of 45.7% and 56% for the west and east terraces.

The breaking up of lenses and stratifications in the Franklin Falls terraces by blasting results in a slight mixture of the material and is important as a means of reducing permeability. Undisturbed samples (after blasting) of sufficient size for tri-axial testing present such a wide variation of grading from point to point that isolation of the more critical uniformly fine materials for testing was found to be infeasible. The results of the blasting compaction at Franklin Falls must be evaluated in part by judgment and experience in critical density testing of fine cohesionless materials similar to those which are of concern in the terrace foundations. The uniform, fine, cohesionless embankment materials at Franklin Falls (which are similar in character and grading to the critical, fine alluvial sands in the terraces) have been tested extensively; and, for the design loadings, a critical voids ratio of 0.67 has been determined. Although not subject to strict correlation, the voids ratios in the terraces, after blasting (with values as low as 0.656 but generally ranging from 0.72 to 0.80), may be favorably compared with those of the mechanically compacted embankment. It is manifest that a comparison of these voids ratios provides considerable assurance that dangers of liquefaction have been reduced greatly.

Decreases in thickness of the loose sand deposits, from 2 to 2.5 ft in a depth of 20 ft, and reduction of the average voids ratio in the natural state of approximately 0.95 to the average values after blasting, previously cited, are indicative that the major settlement hazard has been eliminated. The berms at the toe of the embankment slopes, shown in Fig. 7, serve to contain the foundation; and the deep downstream foundation drain trench, which is backfilled with gravel, provides a ready path for the escape of surplus pore water should the terraces be subjected to vibratory stress. The section of the embankment in the river channel has its foundation approximately 20 ft below the embankment on the terraces, thus virtually enclosing the critical terrace foundations. These facts all provide assurance that adjustments in the terrace foundations, under stress

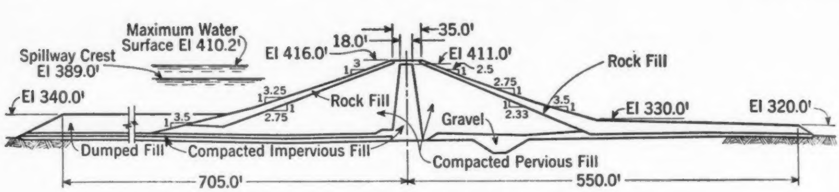

FIG. 7.—EAST TERRACE, FRANKLIN FALLS EMBANKMENT

conditions of reasonable expectancy, will be of a purely local nature and that the possibility of "flow" of the terrace foundations is quite remote.

The cost of compacting the Franklin Falls terrace foundation is approximately \$35,000, or \$0.09 per yd, for the material compacted. This compares with a minimum estimated cost of \$125,000 for treatment by other methods. The equipment used at Franklin Falls to jet the blasting charges in place is shown in Figs. 8 and 9.

FIG. 8.—JETTING DRILL FOR 8-LB CHARGES OF DYNAMITE

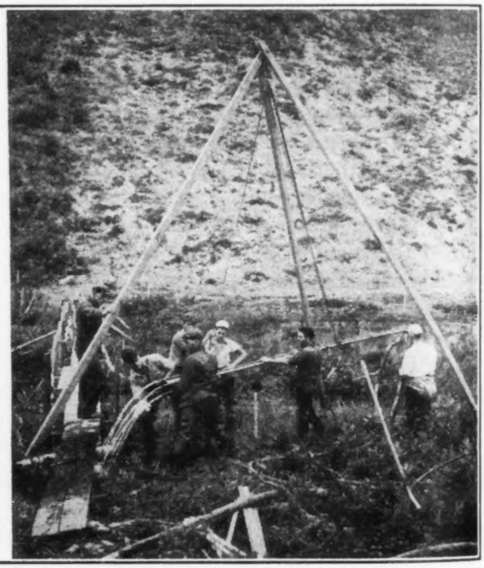

FIG. 9.—JETTING CREW USING DRILL AND TRIPOD

TESTS AT DENISON DAM, TEXAS

At the Denison Dam Site, on the Red River near Denison (see Fig. 10), the river bed is about 900 ft wide and consists of loose deposits of sand and silt, with some intermixed clay extending to depths of 30 ft. During low-water periods the river occupies only a small part of the river bed. Preliminary compaction tests in the South River area, consisting of single charges of dynamite, showed that charges of 16 lb at depths of 16 ft would yield the best results.

A test area 250 ft by 180 ft (North River test area, Fig. 10) was then blasted using seventy-three charges exploded in five individual blasts. An average surface settlement of 0.68 ft was obtained for the area. However, some of this settlement represents a migration of material toward the unconfined river bank.

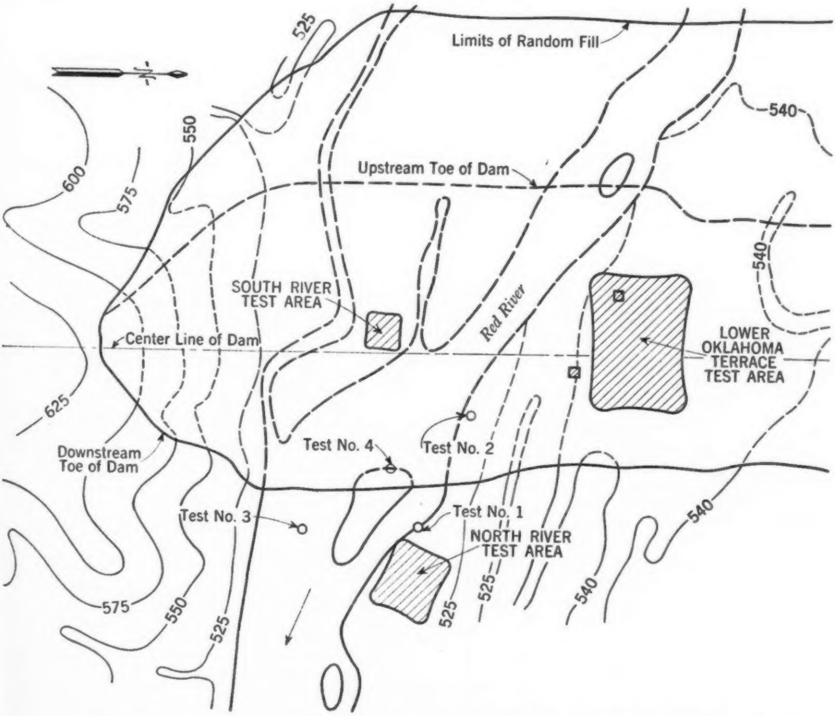

FIG. 10.—RIVER AND LOWER TERRACE AREAS, SITE OF DENISON DAM, IN TEXAS

Tests of undisturbed samples before and after blasting gave the voids-ratio decreases indicated in Fig. 11(*a*).

A second test area on the Oklahoma Terrace (left bank) of the dam where loose, cohesionless materials existed in the embankment foundation was then blasted. The ground-water table in this vicinity is approximately 22 ft below the surface, and pumping and ponding of water were necessary to produce the saturated condition requisite for consolidation blasting. An area 145 ft by 200 ft was enclosed in a small dike and pumping operations were continued with few interruptions for eighteen days with the result that the ground-water table was raised to within 10 ft of the surface; the material between the ground surface and the artificial water table was well moistened. Of the two blasts in the Oklahoma Terrace the first consisted of 312 lb of dynamite split into fifty-one charges placed in three tiers at depths of 5, 10, and 15 ft. The charges in each tier were placed in a pattern of squares 40 ft on a side. Each tier was oriented so that the charges were spaced midway between the charges of the other tiers in plan. The lower tier of sixteen charges of 12 lb of 60% dynamite was set off with instantaneous caps; the middle tier of fifteen charges of 6 lb of 40% dynamite was set off with first-delay caps; and the upper tier of twenty charges of 1.5 lb of 40% dynamite was set off with second-delay caps. The second blast consisted of 414 lb of dynamite divided into fifty-one charges placed in two tiers at depths of 10 and 15 ft.

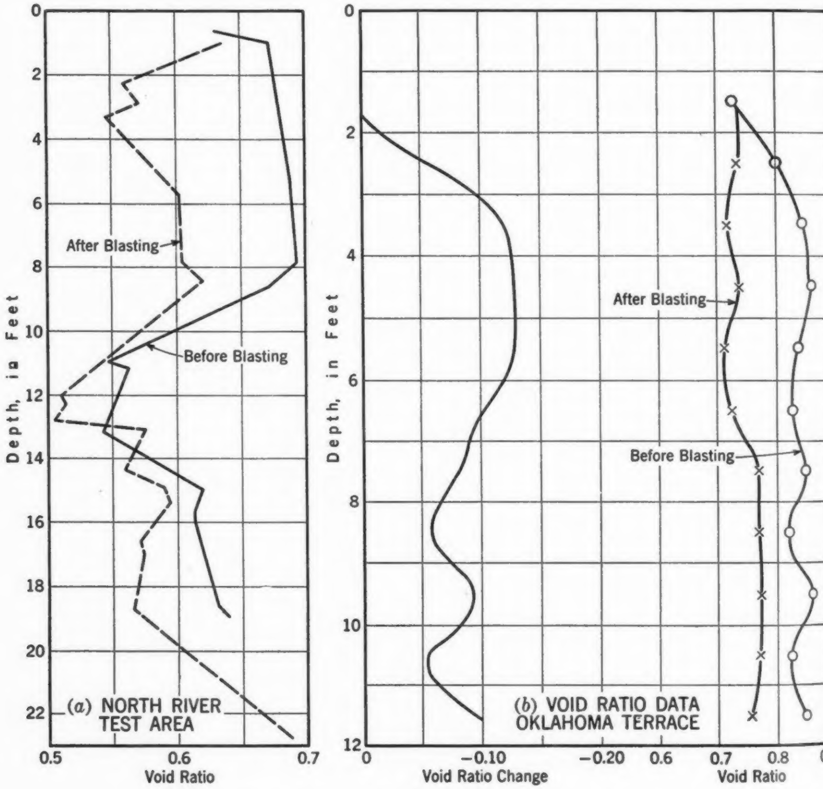

FIG. 11.—VOIDS-RATIO DATA, DENISON DAM, TEXAS

Average settlements of 0.27 ft for the first blast and 0.22 ft for the second blast, or a total of 0.49 ft, were observed. Two test pits, one before blasting and one after blasting, were excavated to a depth of 11.5 ft for obtaining laboratory samples for voids-ratio determinations. A voids-ratio decrease of approximately 0.09 resulted from the two blasts, the major compaction being between the depths of 2.5 ft and 7 ft in the zone where the material was well moistened rather than fully saturated (see Fig. 11(*b*)). The report[3] of the tests at Denison states:

[3] "Inspection Report—Dynamite Compaction Tests, Denison Dam and Reservoir," U. S. Engr. Office, Denison, Tex., dated December 7, 1939 (unpublished).

"Compaction can be obtained by blasting. Results in the River Area show that the critical voids ratio of 0.6 can be very closely approached and that with an average surface settlement of less than one foot. On the Oklahoma Terrace experiments were not carried far enough to learn if the critical voids ratio could be attained, but it is evident that the voids ratio can be decreased considerably even without complete saturation."

Tests at Almond Dam, Hornell, N. Y.

At the site of the proposed Almond Dam near Hornell, N. Y., one of the flood control projects in the Binghamton, N. Y., District of the U. S. Engineer Department, preliminary soils investigations disclosed a deposit of fine to medium sands in the foundation area. A saturated deposit of gravel extended to a depth of 17 ft below the surface and covering the loose, fine sand deposit precluded undisturbed sampling for density determinations of the critical material, except at excessive cost (see Fig. 12). Moreover, it was the geologists'

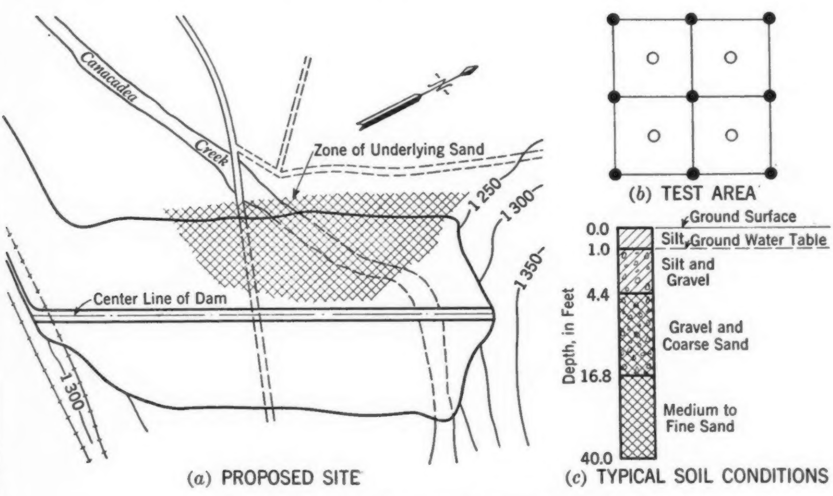

Fig. 12.—Almond Dam, Near Hornell, N. Y.

opinion that the deposit of fine sand was in a fairly dense state. In an effort to confirm this opinion, charges of dynamite were core-drilled through the gravel and into the sand deposit and detonated. The first blast consisted of nine charges of 50 lb each of 60% dynamite. These were placed in a pattern of squares 50 ft on a side, all charges being at a depth of 40 ft below the surface. The second blast consisted of four charges, also 50 lb each in size and at a depth of 40 ft below the surface. The lateral spacing of these four holes was intermediate to the initial nine holes. The third and fourth blasts consisted of reloading and firing the preceding nine-hole and four-hole blasts, respectively. However, the size of each charge was decreased to 20 lb of explosive and the depths were reduced to 30 ft below the surface.

As a result of the four blasts the ground surface of the test area settled an average of 0.1 ft. This was such a slight settlement that it was concluded that the fine sand deposit was initially in a fairly dense state of deposition and no compaction or other treatment would be necessary. It must be emphasized, however, that until more investigations have been conducted in connection with this new method, the failure of an area to settle upon blasting cannot be regarded as conclusive evidence that the material is below the critical voids ratio or that it is stable under all possible stress conditions.

Summary

The method of compacting cohesionless foundation soils by means of explosives was developed and used in the embankment foundation at Franklin Falls Dam. Considerable economy was effected over any previously available methods of compaction, such as vibrators or pile driving, and also over the process of excavating the unsuitable material and recompacting with suitable materials.

The method, in brief, consists of the detonation of charges of explosives spaced laterally and vertically throughout the loose soil mass. Successive coverages of the same area give additional compaction, but the limits of compaction for a specific material by this method have not been investigated. Neither have the tests to date been extensive enough to show whether optimum results are obtained in fully or partly saturated soil.

It can be stated definitely, however, that the present indications are that the method is widely applicable to loose, cohesionless structural foundations provided the materials are substantially saturated prior to blasting. In addition to increasing the degree of compaction of the materials, the horizontal permeability of stratified deposits is greatly reduced.

Acknowledgment

The investigations and tests at Franklin Falls Dam were conducted under the supervision of the writer. Investigations at Denison Dam were conducted under the direction of Maj. Lucius D. Clay, M. Am. Soc. C. E., district engineer, Dension District of the U. S. Engineer Department. Investigations at Almond Dam were conducted under the direction of Lt.-Col. George J. Nold, M. Am. Soc. C. E., district engineer, Binghamton District of the U. S. Engineer Department.

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PAPERS

EXTENSOMETER STRESS MEASUREMENTS, NORTH AVENUE BRIDGE, CHICAGO, ILL.

By Lawrence T. Smith,[1] M. Am. Soc. C. E., and Paul Lillard,[2] Esq.

Synopsis

The design of the North Avenue Bridge (Chicago, Ill.) presented several major structural engineering problems. In developing a rigid-frame design, certain very critical structural assumptions were made which were believed to be quite valid but which lacked definite quantitative substantiating evidence. The importance of the structure justified a systematic extensometer investigation into the actual field-stress condition and structural behavior of the bridge in order to establish the relative validity of these basic assumptions.

As a result of this investigation, the true functional capabilities of this bridge are now known, definitely and accurately, free from the shadow of doubt that often clouds the reflections of the structural designer in his application of pure mathematical logic to the complex factors affecting the higher type of structure.

Field Measurement of Stresses

The first recorded instance of the actual field measurement of stress in an existing structure by means of the extensometer was in 1883 when J. J. Frankel described his investigations (12)[3] upon the primary stresses in a Pratt truss bridge. In 1899 M. Mesnager published (27) an account of extensometer stress measurements made upon a railroad bridge. In 1901 M. Rabut described (34) stress measurements on girder and truss railroad bridges. In 1907 the American Railway Engineering Association (A.R.E.A.) conducted stress measurements of limited scope on girder and truss spans to determine relative impact allowances for design purposes (21).

In all of these investigations, which constitute probably the entire scope of the work done in this field until about 1917, no stress measurements were made

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 15, 1941.**

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[3] Numerals in parentheses, thus: (12), refer to corresponding items in the Bibliography (Appendix).

to compare computed and measured stress or to determine dead-load stress; and no measurements were made on statically indeterminate structures. Such work as had been done was greatly restricted to the determination of live-load stresses and to the evaluation of relative impact. Many of the results contained serious instrumental errors. The use of the extensometer in the systematic investigation of true field stress and the verification of structural design theory applied to an existing structure was of no consequence until 1917 when D. B. Steinman, M. Am. Soc. C. E., conducted his extensive systematic stress measurements on the Hell Gate Bridge (39). This work was done on a monumental scale and remains today probably the most comprehensive stress survey ever made upon a bridge in service.

In the interval from 1917 to date (1941) there has developed an increasing interest in the extensometer as a precision instrument of value to the structural engineer; but the period has been characterized by few stress measurements on structures under construction or in service. The use of the instrument has been confined mainly to investigations upon small models; upon structures erected for specific test purposes, and subject to structural control; and upon structures or members observed under laboratory conditions. In view of the means at hand it is remarkable that the phenomenal progress made in structural design theory in the two decades 1920–1940 has not been accompanied by a wider utilization of the extensometer by structural engineers.

Field stress measurements are expensive, and they are time-consuming and laborious. They require a high order of instrumental and technical skill in order to be of value. Few organizations have the personnel equipped for this work and few can afford the heavy financial outlay for the apparently intangible results achieved. However, there are undoubtedly many situations that arise in practice where the use of this valuable tool would result in a great economy of effort and easily would secure for the engineer the answer to many otherwise impossible stress problems.

Description of Bridge

The Project.—The North Avenue grade-separation project of the Chicago Park District was initiated to provide adequate pedestrian and vehicular grade-separation and traffic-interchange facilities for the introduction of vehicular traffic from the improved LaSalle Street into the Outer Drive of the Chicago Park District, in the southern area of Lincoln Park (see Fig. 1). The project required a complete redesign and reconstruction of most existing facilities over a large area of the park, involving landscaping and the relocation of roadways and walks, sewers and electric lines, and the construction of several large pedestrian subways and a large, skewed, rigid-frame type of vehicular bridge constituting the main unit of the project.

This bridge is designed to accommodate the extremely heavy Outer Drive traffic passing over the likewise heavy northbound LaSalle Street flow. Both streams are interchanged at a junction immediately north of the bridge. Because of space limitations, it was necessary to cross the two streams at a sharp

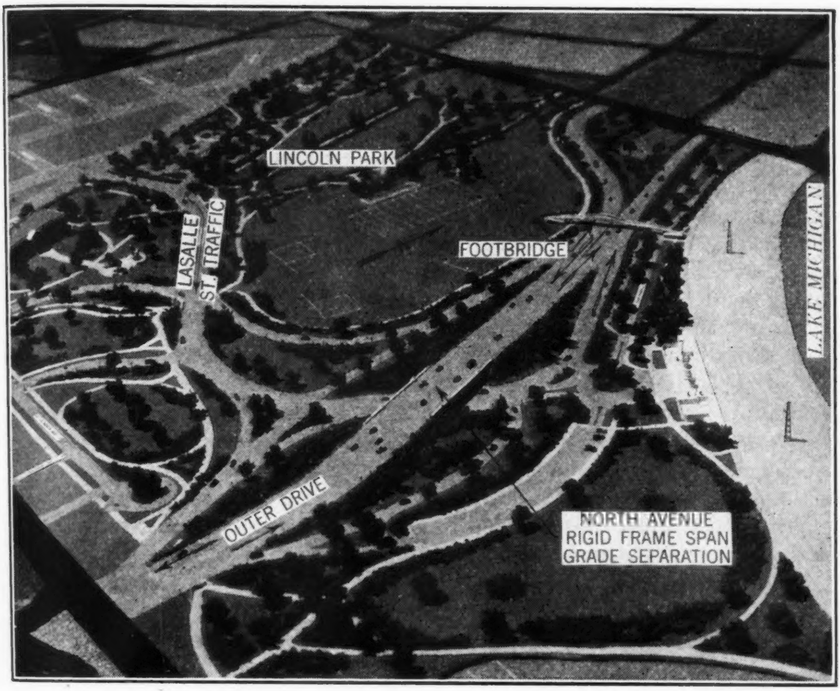

FIG. 1.—MODEL SHOWING LAYOUT OF PROJECT

skew. A high water table prevented an adequate depression of the lower roadway, and the resultant gradient problem placed a definite limitation upon the allowable deck thickness, indicating the use of a structure of the rigid-frame type.

The North Avenue Rigid-Frame Bridge.—The immediate surface stratum at the site consists of a fine lake sand of variable thickness resting upon several strata of glacial till composed of blue-brown clay. Core borings taken at the site showed the upper stratum of this till to be a very wet, soft, plastic, colloidal clay apparently unreliable for major foundation purposes. Load tests made upon this clay showed it to be of little value in supporting a rigid-frame structure. Mantle rock, a reliable Niagara dolomitic limestone of great depth, lies at about 90 ft below Chicago Datum at the bridge site. After considerable study, in which the use of piling was considered, it was decided to rest the structure on cylindrical concrete subpiers driven to mantle rock or to the cemented gravelly claypan which lies immediately above rock. Studies having indicated the existence of much heavier hinge thrusts than could be resisted alone by these subpiers, the lower roadway slab was reinforced and designed to function as a tie in restraining the abutment hinges.

The superstructure is a skewed, two-hinged, reinforced-concrete, rigid frame with deck and abutments of slab-and-girder design and with hinges tied by the reinforcement of the lower roadway slab. The bridge loads are transmitted to rock through spirally reinforced concrete subpiers. The deck is made of slab-and-girder design primarily for economy, but also because of the practical impossibility of predicting stresses in a skewed, rigid-frame bridge of the slab or barrel type. The deck at the crown is of conventional T-beam construction; at the haunches, because of the existence of heavy negative moments, it is of cellular construction with a lower flange slab for the resistance of the bottom fiber compressive stresses.

The skew angle between deck girders and abutments is 55°. The clear span of deck girders is 101 ft and the centroidal span of the frames is 105 ft. The ratio of centroidal rise to centroidal span is 1 : 7. The width of bridge along abutments is 130 ft, and the curb width of the upper roadway is 102 ft.

Structural Design and Stress Analysis.—Supporting subpiers are spirally reinforced-concrete columns of the Chicago caisson type driven to hardpan or rock by a patented rotary process. These subpiers were designed only for axial compression as short reinforced-concrete columns, the diameters being selected for economy and a minimum inspection space. The rotary process used in sinking these piers eliminates the usual large working space requirement of the conventional Chicago caisson and allows the use of small diameters to great depths since excavation is almost completely mechanized. Lateral bending was not provided for in proportioning these subpiers beyond that available in the axially designed section. The tie-slab (Fig. 2) was considered the active agent in resisting hinge thrust, any residual bending being taken by the passive resistance of the substrata. On this basis the tie slab was proportioned to accommodate the entire hinge thrust, unrelieved by active or passive back-fill pressures and subpier restraint. In making the design it was not believed that the effect of active or passive back-fill forces would be large, either upon hinge thrusts or upon the stress functions of the superstructure. Sleeves for mud-jacking were provided in the tie-slab for the readjustment of the slab in the event of subsequent deleterious settlement.

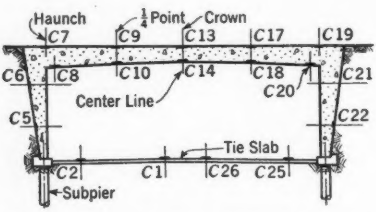

FIG. 2.—TYPICAL SECTION, SHOWING LOCATION OF STATIONS ALONG CENTER GIRDER

The bridge was designed to carry the Park District standard truck live load, consisting of a file of two 24-ton trucks surrounded by a uniform load of 100 lb per sq ft of deck. Impact was taken at 22%. The maximum temperature differential between superstructure and tie-slab was taken at 25° F, and the effect of hydration shrinkage was based upon a coefficient of 0.0002. No beneficial effect was considered to be derived from active or passive backfill pressures.

Preliminary proportioning was made by means of the moment-distribution method (7), assuming a two-hinged frame of variable moment of inertia. Final analysis of this preliminary proportioned structure, after complete structural detailing, was made by means of the elastic theory, using two separate methods of attack by two computers: (1) Constant $\frac{ds}{I}$, and (2) constant ds. Each analysis was thorough, and the use of the two methods furnished an excellent check. Completion of the analysis by elastic theory showed a very close agreement with the preliminary functions obtained by the moment-distribution method. The bridge was treated for analysis as a series of interior girders or frames, and fascia girders or frames. Each was analyzed separately. As actually analyzed, the frame was assumed to be indeterminate only in the first degree, since the only decisive unknown is the horizontal component of the hinge thrust. The effect of subpier restraint and backfill effort was considered negligible in influencing the hinge thrust and superstructure functions to any significant degree. The horizontal component of the hinge thrust is a function of the properties and loads of the superstructure and the deformations occurring in the tie slab. The equation for hinge thrust under a vertical loading upon the deck is

$$H_0 = \frac{\int_L^R \frac{M' \, y \, ds}{l}}{\int_L^R \frac{y^2 \, ds}{l} + \frac{L}{A_s}} \qquad (1)$$

in which $\frac{L}{A_s}$ represents the restraint of the tie slab. Design studies showed that, theoretically, the effect of the tie slab upon the computed stress functions of the deck is practically negligible, and that small horizontal movements in the hinges produce negligible moment variations in the superstructure. The truth of this statement, in practice, has been established definitely by the results of the extensometer stress study. As measurements during the test showed that there was no movement of the hinges due to applied loads, no elongation of the tie rods was taken into account in evaluating the results. However, the actual design proportioning of the frames was based upon consideration of the tie-slab restraint.

Early stress studies showed the negative moment compression at the haunches to be very high, necessitating the use of a lower flange slab for inverted T-beam action and resulting in the cellular construction of the deck from quarter point to abutment. This cellular part (Fig. 3) was detailed with a taper in the lower flange slab starting at the quarter point and widening out to full width at the abutment. As finally treated, it produced a very satisfactory architectural motif strongly suggestive of greater span and strength.

Shears in the deck girders are high and are accommodated by bent bars and by vertical stirrups. For the accommodation of leg shears, which (according to recent laboratory investigations) are uncertain in actual intensity and are probably much higher than the computed values, more shear reinforcement was used than was indicated by conventional theory.

The abutment hinges were detailed to allow a fairly free rotation with a minimum of moment transmission into the subpiers. The subpiers, however, move horizontally in conjunction with the tie-slab and the abutment hinges. The tie-slab is anchored positively to the lower legs and the hinges. A heavy subpier cap girder was provided to effect a lateral distribution of unequal local loads to the subpiers, and a like provision was made for the distribution of local loads to the lower part of the abutment legs by the thrust girder.

The effect of skew was neglected in analyzing and proportioning the interior frames, although it is obviously a factor in the action of the fascia and adjacent outer girders under certain asymmetrical loadings. However, it was deemed satisfactory to proportion the frames by conventional methods. The wisdom of this course has since been proved by the results of the stress measurements made upon the completed structure.

FIG. 3.—MAIN-FRAME DECK GIRDERS BEFORE CEMENT FINISHING, SHOWING CELLULAR HAUNCHES AND EXTENSOMETER STATIONS

The total weight of reinforcing steel used in the superstructure (Fig. 4) and tie slab was 374 tons, and the total volume of concrete was 2,650 cu yd, a volumetric proportion of 2.1%. The setting of steel presents a difficult problem because of the weights and lengths involved. However, this percentage of steel (2.1%) is very reasonable, compares very favorably with that of other structures, and indicates economy in the use of this type of bridge.

Construction.—In sinking the subpiers it was necessary to develop a technique especially adapted to the peculiar subsurface conditions and the rotary process used. A primary requirement in building a structure of this type is the proper design of an unyielding centering and its skilful erection. In this kind of structure, in which few lines are straight, a high quality of workmanship on the part of the carpenter is of vital importance.

Although the structure was built with Park District forces, a close field engineering control was established very early for purposes of securing a uniform production of concrete, satisfactory control-test cylinders for use in the stress measurements, and a qualified inspection service during the construction period.

Control-test cylinders were made from each pour and were subjected to breaking tests to establish the quality of the concrete. A number of these cylinders were given stress-deformation tests to secure the stress-deformation

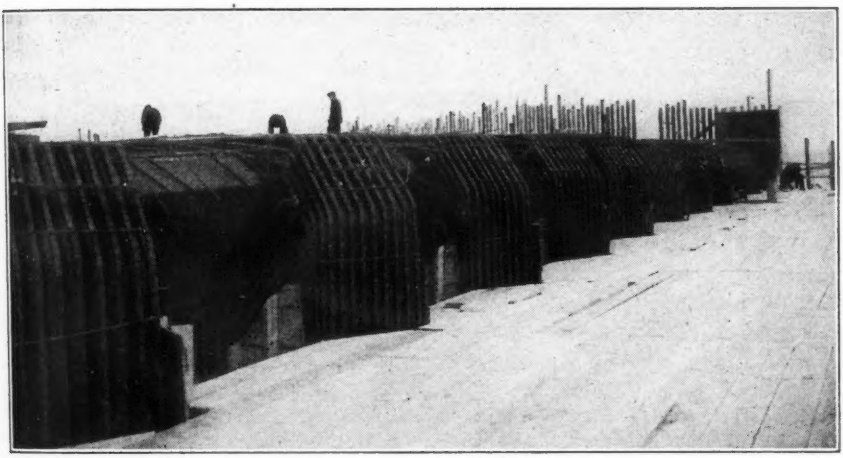

FIG. 4.—VIEW SHOWING REINFORCEMENT AT THE HAUNCH

diagram below the failure point. This was likewise done for specimens of reinforcement cut from the bridge. The results were used in selecting the value of $n = \frac{E_s}{E_c}$, which was taken as 10.

EXTENSOMETER STRESS MEASUREMENTS

Object.—The massive character and the importance of the structure were two of the several reasons why it was considered advantageous to conduct an extensive series of strain-gage extensometer measurements: (1) To ascertain the true stress condition as a matter of safety, (2) to make a comparison of true and theoretical stresses as a matter of structural engineering research, (3) to study the elastic action of the system and the behavior of the bridge under service conditions, and (4) to secure data of value in the future design of similar bridges.

Preparation of Stress-Gaging Stations.—For the measurement of stresses twenty-eight strain-gage stations were provided for in the design and were built into the deck girders, tie-slab and abutment legs of the center-line interior frame, and the east fascia frame (see Fig. 3). At each station the outer face of a length of reinforcing bar carrying computed stress was formed, partly exposed for observation; a thermometer pipe was embedded to the center of the concrete mass; and a pair of thermocouple leads were embedded for the potentiometer determination of internal concrete temperature.

An extensometer designed especially for this study (see Fig. 5) was used most of the time. The instrument consists of a channeled brass bar in which two snugly fitted brass slides move, each holding a sharply pointed center punch. The one slide is clamped securely in position and remains fixed during any given series of measurements. The other slide, being adjustable and actuated by a knurled nut and threaded shaft, is the movable element of the instrument; and its extreme end bears against, and actuates, a strain-gage dial. There are no gears, levers, or bearings, and there are a minimum of parts in this extensometer. Error from mechanical sources is eliminated almost entirely since the movable slide is operated in a snugly clamped condition. Observations are taken upon extremely fine punch-mark holes or gage lines scribed with center punches made of hard alloy drill steel rigidly attached, 20 in. apart, to a steel standardizing punch bar or trammel. The gage points or lines are scribed upon highly polished areas of the reinforcing bar; they are made by a very light pressure of the hand, producing a line as small and as fine as possible consistent with visibility under a strong magnifying glass.

When a stress-gaging station was originally established, two areas of the reinforcing bar, 20 in. apart, were ground smooth and given a very high mirror-like polish with very fine emery paper and jewelers' rouge. The standard, trammel punch bar, maintained at a temperature of 32° by immersion in ice water, was used to establish the gage lines. Two punch holes were made, the one receiving the fixed element of the instrument being made somewhat larger than the punch hole used for adjusting the dial in making readings. The latter hole was made extremely small, and its use was primarily as a reference point, although accurate observations could be made upon it. Actual readings were taken upon the extremely fine gage lines scratched across the polished face of the steel reinforcement.

Stress measurements were made by a team of two structural engineer observers assisted by two or three nontechnical helpers who prepared stations, held flashlights, and carried on the skilled mechanical work required in cutting bars, arc welding, polishing, grinding, building scaffolding, and handling test loadings. These helpers were usually skilled carpenters or ironworkers. In Fig. 5, the strain-gage dial, the reading end of the instrument, and the gage line being observed are at the observer's right hand. The alternate observer is holding the fixed point in its position. The standard punch bar for scribing lines rests on the tool box.

Instrumental work of this nature, conducted under widely varying temperature conditions, requires a base for length comparison that is practically unchangeable under ordinary temperature variations. For this purpose a square, forged bar was used, which was made of a high nickel-iron alloy with a coefficient of temperature expansion of 0.0000004 to 0.0000005, which is about one sixteenth that of structural bridge steel. One face of this bar was brought to a mirror-like polish. Gage points and lines were established on this polished face for each series of stress measurements. This was done with the reference bar and the trammel punch bar maintained at 32°. The function of these gage lines on the reference bar was to provide a base of unchangeable length to which the extensometer could be applied in each reading and given its initial zero setting. By application of the extensometer to similar lines established on the reinforcing bar under observation, a direct comparison of length is made and the true difference or deformation is indicated on the gage dial. For record

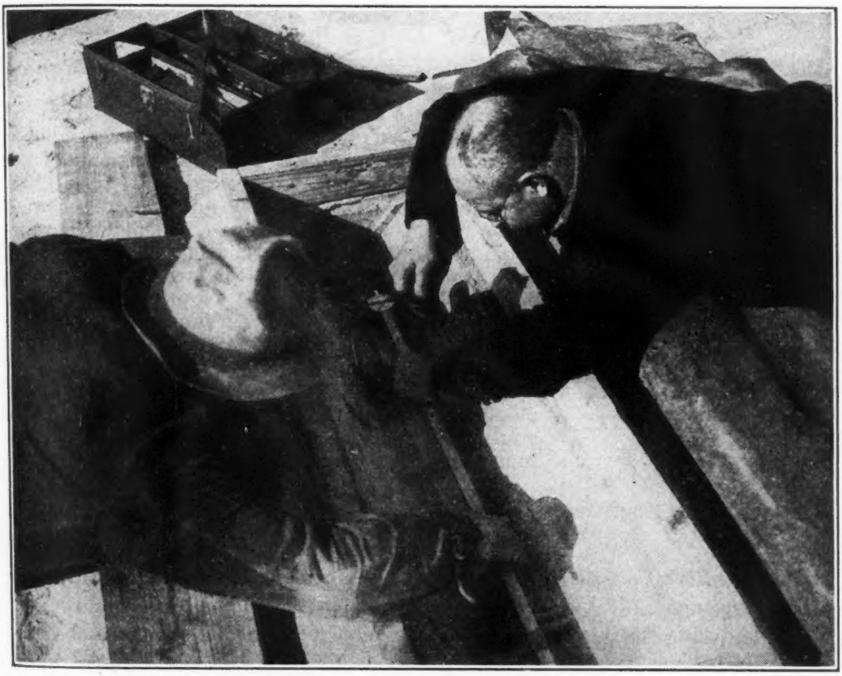

Fig. 5.—Making Strain-Gage Measurements with Special Extensometer

and identification each set of gage lines was circled with colored paint, different colors being used for the various loadings and stress series. The reference bar, as well as the standard trammel punch bar, was kept in ice water throughout any series of stress measurements. It was found that, with a little care, the temperature changes in the brass extensometer bar could be minimized. As a matter of fact, the short time that ordinarily elapsed between index reading and index check reading upon the reference bar furnished an excellent check on any temperature errors originating in the extensometer. The use of the thermocouple installation was found to be time-consuming, and the main reliance was placed upon the use of a large number of accurate mercury thermometers. All stress readings were corrected for temperature variation by reduction to a common station temperature, assuming a free adjustment of the reinforcing to thermal change without the development of temperature stress. This is not strictly true, however, and does not entirely eliminate the temperature factor from measured stresses since the corrected deformations contain a temperature increment that it is quite impracticable to evaluate. This increment for temperature varies to a maximum of about 15° and is very small and negligible in amount, so that assuming it to be zero produces deformation readings within a very satisfactory range.

Scope of Extensometer Stress Measurements.—Extensometer stress measurements were made upon the structure under the following conditions of loading:

Load	Description
Dead Load	Structure under its own dead load, backfill pressures, and restraint from subpiers.
Test Live Load I	A transverse test load of 65 tons uniformly distributed along the crown of the bridge on the skew parallel to abutments, producing a concentrated load of 6.6 tons at the crown of each deck girder.
Test Live Load II	A transverse test load of 105 tons uniformly distributed along the crown of the bridge on the skew parallel to abutments, producing a concentrated load of 8.1 tons at the crown of each deck girder.
Test Live Load III	A longitudinal test load of 79 tons placed directly on the center-line interior girder and uniformly distributed to simulate the conventional truck live load used in design.
Test Live Load IV	A transverse test load of 160 tons uniformly distributed along the crown of the bridge on the skew parallel to abutments, producing a concentrated load of 13.8 tons at the crown of each deck girder.

Behavior of Structure After Removal of Centering and Under Temperature Changes

Temperature Changes in the Structure.—A systematic record was made of hinge movements, deck deflections, and temperature changes in air and bridge concrete during the period of the investigation. The results of these observations furnish an interesting portrayal of elastic condition and present much valuable evidence for certain conclusions concerning the actual physical behavior of the structure.

Temperature observations on air and concrete were made in great detail over a period of about four months, embracing a mean high concrete temperature of 78° and a mean low concrete temperature of 16°, a mean high air temperature of 96° and a mean low air temperature of 15° below zero. When extensometer observations were actively under way, periodic readings were taken at all stress-gaging stations repeatedly during the day in order to furnish a reliable basis for the temperature adjustment of stress measurements.

The result of the observations on air and concrete temperatures is plotted as shown in Fig. 6. No attempt has been made to smooth the curves. The points as plotted in Fig. 6(*a*) represent the mean temperature observed during daylight hours. No temperature observations were made at night so that the physical effect of nocturnal changes is not shown in the temperature curves. Because of this fact, and because the value plotted is actually the daily mean temperature, the lag of concrete temperature behind air temperature is not clearly shown in some instances.

An examination of the air and concrete temperature curves shows the very quick response of the structure to air-temperature changes. This response is very much quicker than is customarily assumed. With air temperatures ranging from plus 32° to plus 95°, the largest observed difference between concrete and air was only 20° and only for a short period of time (six hours). Except for occasions of sudden change, the temperature of the bridge concrete follows very closely that of the adjacent air. Subsidiary local effects were

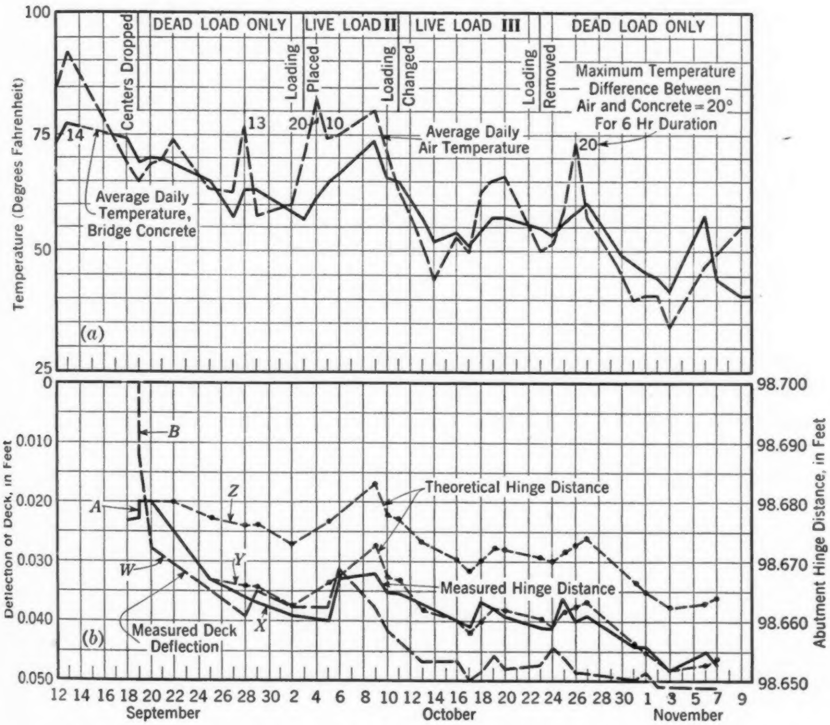

FIG. 6.—DAILY TEMPERATURE VARIATION, MOVEMENT OF HINGES, AND DEFLECTION OF DECK

observed—the greater warming of fasciæ under summer sunlight, their greater cooling under strong lake gales, and the quicker response of crown and the slower response of abutments. The mean temperature of the tie-slab coincided closely with that of the superstructure, the greatest daily difference being only 3°. With air temperatures below the freezing point and as low as 15° below zero, however, the thermal effect of the heat retained in the larger backfill mass produced a response in the bridge different from that observed for air temperatures above the freezing point. Thus, when the air temperature dropped to 15° below zero and continued at that low level for two days, remaining below

10° below zero for about seven days, the temperature of the deck dropped only to 15° above zero and remained at that level throughout the low period until the air temperature had risen to above freezing. This action is to be expected because of the slow penetration of frost into the backfill mass. Until this soil is generally frozen, there will be a definite heat transfer from the relatively warm backfill to abutment legs and deck, resulting in the dampening action that is observed in the structure in sub-freezing temperatures.

Hinge Movements and Deck Deflections.—The investigation of the hinge movements was based primarily upon daily precision measurements of the longitudinal horizontal distance between the abutment hinges of the center-line interior frame. No observations were made upon hinge rotations. All measurements were corrected for temperature, tape sag, and tape tension, and were reduced to 32° for study. As an adjunct in ascertaining the origin of the day-to-day hinge displacements, the theoretical hinge lengths of the deck under the same temperatures were computed, reduced to 32°, and plotted in conjunction with the actual measured hinge-length curve. Two of these theoretical curves were actually computed—one based upon the conditions at the time centers were dropped, and the other based upon conditions that obtained at the time the structure reached apparent equilibrium about six days after the removal of the centering.

In Fig. 6(*b*): Curve X is the actual adjusted measured distance between gage lines on the abutment hinges; curve Y is the theoretical computed distance between deck gage points corrected to temperature on September 25 when equilibrium was reached; curve Z is the theoretical computed distance between deck gage points corrected to the temperature on September 19, when the centers were dropped (increment A at the beginning of curve Z represents the increase in hinge distance, due to the dropping of the centering, and the resultant dead load on the structure); and curve W is the actual measured crown deflection of the center-line girder. Increment B, at the beginning of curve W, represents the deflection at the time the centers were dropped.

The investigation of vertical deflections in the deck was based upon daily precision measurements at five stations established along the crown of the bridge. These stations consisted of carefully built deflection gages that were carefully scribed prior to dropping the centering. A deep concrete post was sunk through a hole in the tie-slab at midspan under the center-line frame for the purpose of providing precision points for checking the results of observations on the deflection gages. A brass plug in the girder overhead and a similar plug in the independently founded post provided these precision points. These deflection observations were likewise reduced to 32° for study.

Readjustment Period Following Removal of Centers.—A study of the curves of observed hinge distance, observed deck deflection, and observed temperature variations indicates the existence of a distinct period of structural readjustment of about six days duration following the removal of the deck centering before the structure reached equilibrium under its own dead load.

At the time centers were struck, this heavy timber framework supporting the deck had been in place eighty-one days under the full load of the deck. It transmitted this load, in turn, to the tie slab upon which it rested. The superstructure, tie slab, and subsoil supporting the tie slab had been in equilibrium under this load condition for many weeks. Striking the deck centering removed the heavy vertical loads which had rested upon the tie slab and which had produced a compression in the subsoil. This subsoil is of poor structural character. Bearing tests made prior to design showed it to be very plastic and low in restoration properties. It is probable that the arched crown of the tie slab facilitated the development of appreciable outward thrusts upon the abutment hinges under the action induced by the plastic condition of the subsoil under the centering load. These thrusts apparently caused an outward movement of the hinges prior to striking of the deck centers, and the system had stabilized in this condition.

Striking of the deck centers removed the heavy centering load from the tie slab, but the plastic nature of the subsoil precluded anything other than a slow recovery of subsoil and tie slab to the unloaded condition. The tie slab recovered slowly, attaining its dead-load equilibrium only after about six days of movement. During this period the hinges drew inwardly at a much greater rate than was indicated theoretically by temperature change. After equilibrium was reached, the horizontal movements in the hinges were almost entirely of temperature origin and were caused by variations in the length of the bridge deck under changes in temperature. The observed day-to-day movements in the hinges are in very close agreement with the theoretical values computed for temperature changes only.

Conclusions Concerning the Behavior of the Bridge.—The following conclusions seem warranted as a result of the study made upon the behavior of the bridge under temperature change and at the striking of centers:

(1) There was a definite readjustment period of about six days, which followed the removal of the deck centering, before the structure attained static equilibrium. During this period the hinges moved inwardly at a greater rate than theoretically computed for the temperatures observed during the period. This phenomenon is traceable to the retarded recovery of the subsoil supporting the tie slab, following the removal of the deck centering load which it formerly sustained. When final equilibrium was reached, at the end of this readjustment period of six days, the structure behaved elastically and geometrically as predicted by theory.

(2) After structural equilibrium is reached following the period of readjustment, the abutment hinges move horizontally in very close response to temperature changes in the superstructure. There is a very close agreement in the curves of theoretical and observed hinge distance, indicating that nearly all of this movement originates in length changes in the deck under temperature variations.

(3) After structural equilibrium is reached, the effect of the restraint of the subpiers and the passive resistance of the approach backfill is apparently quite negligible in modifying the action of the hinges.

(4) The very close agreement of the curves for theoretical and measured hinge distance indicates a probable low magnitude for temperature stresses in the structure. This is further established by the results of the extensometer measurements.

(5) Above the freezing point the internal temperatures of the bridge concrete respond very quickly to variations in air temperature. Below the freezing point the retained heat in the unfrozen backfill soil mass exerts a distinct dampening action on changes in temperature in the superstructure, retarding very greatly the response of concrete temperatures to sub-freezing air temperatures.

Comparative Study of Stress Measurements

Extensometer Measurements Under Dead Load.—The design provided for a crown key to reduce the accumulated shrinkage stresses. This key was poured thirty days after the first deck pour was made. Abutment backfilling started several weeks later, and upon its completion the deck centering was struck—eighty-one days after the pouring of the crown key. The behavior of the deck centering at the time it was struck, the circumstances under which the dead-load stresses were measured, and the close agreement of dead-load stresses measured by standard extensometer procedure and by the cut-bar technique indicated strongly that the centering was still carrying a very considerable part of the weight of the deck.

The measured and theoretical computed stresses under dead load show surprisingly close agreement. For purposes of comparison between measured and theoretical conditions, the measured stresses were used in computing equivalent resisting moments. Compression stress readings in most instances ran excessively high, a condition directly traceable to flexure in the station reinforcing bar in the exposed length that was partly or entirely freed of bonding concrete. The results given in this paper are based accordingly upon the observed tensile stress in the steel and the theoretical linear location of the centroidal axis, assuming no concrete in tension. There is a strong probability of the concrete taking tensile stress, which might tend to modify the measured moments; but it was deemed more practicable to assume the tensile-free condition commonly used in design. The standard linear flexure equation for compression-reinforced T-beams was used in deriving the equivalent measured moment from the measured stress. The moment of inertia of transformed sections was used. The use of a parabolic flexure equation in computing measured resisting moments was given some study but was not deemed practicable.

In order to determine the total combined stress from dead load, temperature change, hydration shrinkage, backfill effort, and subpier restraint (which existed in combination after striking of the deck centering), and to secure a check upon the dead-load stresses that were measured at the time the centering was struck, stress measurements were made upon a number of bars, using the cut-bar technique which the writers developed to facilitate the solution of this special problem. This cut-bar technique involved:

(*a*) Careful removal of the concrete cover from a reinforcing bar carrying computed stress;

(*b*) Preparation of gage lines on the reinforcing bar and on the reference bar;

(*c*) Readings for index reading before any further bonding concrete is removed;

(*d*) Careful removal by hand chipping of all bonding concrete between gage lines;

(*e*) Intermediate reading for deformation;

(*f*) Careful cutting of the bar outside the gage length in a manner causing no temperature changes in the bar;

(*g*) Final reading for deformation in the no-stress condition; and

(*h*) Arc-welding the severed ends of the bar to restore some part of the structural value.

Stress measurement with the cut-bar technique was done in such a manner as to show the true, total, unmodified stress of all origins existing in the bar at the time of measurement. Stresses measured by this technique are entirely free from the error due to imperfect temperature corrections. The series of measurements made by this method furnished an excellent check on the measurements made by the standard dead-load procedure. It is believed that the cut-bar technique is essentially a new departure and certain to be of value in future studies involving extensometer stress measurements.

A study of the data secured in the stress measurements upon the center-line interior frame is summarized in Table 1. The significance of the various column headings in this table may be clarified further, as follows:

Col. No.	Description
2	Extreme temperature range at the station during stress observations
3	Arithmetical mean of observed dead-load deformations at the station, corrected for temperature
	Stress in the Steel Reinforcement, Computed by Means of:
4	Young's modulus
5	Elastic theory
	Bending Moments at the Station:
6	Theoretical
7	Measured resisting moment, computed from the measured tensile stress and an assumed centroid

A graphical comparison of computed and measured stresses and moments is shown in Figs. 7 and 8. On these curves:

C denotes computed theoretical stress or bending moment for the loading indicated;

M denotes the measured stress (Fig. 7) secured by means of a standard extensometer, or the equivalent measured resisting moment (Fig. 8), using the measured tensile stress and a theoretical centroid; and

MS denotes the measured combined stress (Fig. 7) secured by the cut-bar technique, or the equivalent measured resisting moment (Fig. 8), using the tensile stress and theoretical centroid in the cut-bar technique.

TABLE 1.—COMPARISON OF MEASURED AND THEORETICAL STRESS FUNCTIONS

Stress gaging station (see Fig. 2)	Temperature range (degrees F)	Mean measured corrected deformation	Stress in Reinforcement (Lb per Sq In.)		Bending Moments in Hinged Frame (Ft-Lb)		Ratio, Col. 7 / Col. 6
			Equivalent measured	Theoretical	Theoretical	Equivalent measured	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(*a*) Dead Load; Interior Frame							
C 7	78–78	0.0054	8,130[b]	8,620	−2,528,000	−2,388,000	0.95
C 7[a]	26–26	0.0054	8,130[b]	8,620	−2,528,000	−2,380,000	0.94
C 8	78–78	−0.0049	−7,350[c]	−3,790	−2,528,000	....	....
C 9	55–68	0.0017	2,650	680	−130,000	−342,000	2.65
C 9[a]	30–30	0.0009	1,470[b]	680	−130,000	−216,000	1.66
C10	71–71	−0.0009	−1,360[c]	−1,745	−130,000	....	....
C13	55–78	−0.0034	−5,100[c]	−2,145	557,000	....	....
C14	71–76	0.0067	10,100	8,430	557,000	665,000	1.19
C14[a]	26–26	0.0049	7,350[b]	8,430	557,000	490,000	0.88
C17	71–76	0.0010	1,500	680	−130,000	−220,000	1.69
C17[a]	30–30	0.0025	3,750	680	−130,000	−247,000	2.50
C18	71–78	−0.0034	−5,100[c]	−1,745	−130,000	....	....
C19	60–71	0.0055	8,300[b]	8,620	−2,528,000	−2,438,000	0.96
C19[a]	71–78	0.0051	7,700[b]	8,620	−2,528,000	−2,268,000	0.90
C20	27–27	−0.0046	−6,900[c]	−3,790	−2,528,000	....	....
(*b*) Test Live Load III; Interior Frame[d]							
C 7	42–56	0.0024	3,600	5,160	−1,515,000	−1,074,000	0.71
C 8	40–57	−0.0025[c]	−3,750[c]	−2,230[c]	−1,515,000	....	....
C13	43–56	−0.0036[c]	−5,500[c]	−1,800[c]	−459,000	....	....
C14	40–56	0.0033	5,000	7,020	−459,000	335,000	0.73
C19	41–58	0.0010	1,500	5,160	−1,515,000	−478,000	0.32
C20	40–50	−0.0016[c]	−2,350[c]	−2,230[c]	−1,515,000	....	....
(*c*) Live Load IV; Interior Frame[e]							
C 7	17–24	0.0008	1,150	1,270	−373,000	−328,000	0.88
C 8	17–23	−0.0029[c]	−4,360[c]	−550	....	....	....
C14	16–27	0.0022	3,220	3,815	241,000	204,000	0.92
C19	18–25	0.0010	1,500	1,270	−373,000	−437,000	1.17
C20	17–24	−0.0029[c]	−4,360[c]	−550	....	....	....
(*d*) Dead Load; Fascia Frame[f]							
F14[g]	55–60	0.0040	6,000	9,700	....	....	0.62[h]
F17[g]	54–61	−0.0005	−750	1,000	....	....	0.75[h]
F19[g]	55–62	0.0050	7,500	10,700	....	....	0.70[h]

[a] Cut. [b] Denotes that the value of the tensile stress is checked very closely by cutting the station bar. [c] Denotes that the value given is probably in error due to compression flexure in station bar. [d] Test Live Load III was a test loading of 79 tons of sacked cement placed directly on the center-line interior frame to simulate the conventional Park District truck live loading. [e] Test Live Load IV was a test live loading of 160 tons of sand uniformly distributed along the crown of the bridge on the skew parallel to abutments producing a concentrated load of 13.8 tons at the crown of each deck girder. [f] Cut-bar technique was not applied to measurements on fascia frame. [g] Stations located on south half of fascia frame only; north half not established; station numbers correspond to those of similarly located stations in interior frame.
[h] Ratio, $\frac{\text{Col. 5}}{\text{Col. 4}}$.

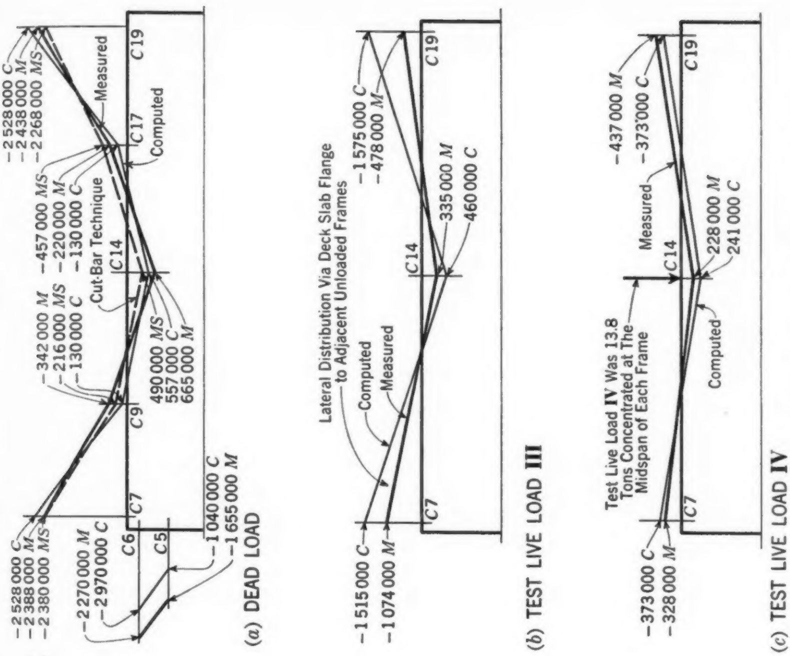

FIG. 8.—COMPARISON OF THEORETICAL AND MEASURED MOMENTS (INTERIOR FRAME)

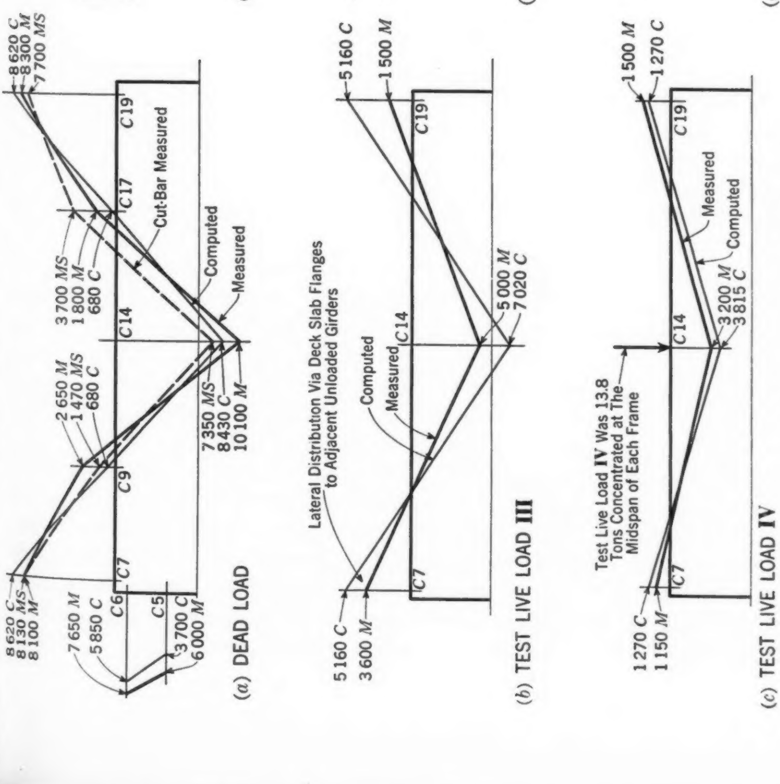

FIG. 7.—COMPARISON OF THEORETICAL AND MEASURED STRESSES (INTERIOR FRAME)

Stations C 7 and C 19 were at the face of the abutment; Station C 14 was at the midpoint of the frame; and stations C 9 and C 17 were at the quarter points. No readings were taken at stations C 9 and C 17 during test live loads III and IV.

The following conclusions seem warranted by the data secured in the measurements under dead load (see Figs. 7(*a*) and 8(*a*)):

(6) Under dead load the structure is stressed very close to the theoretical values computed by means of elastic theory.

(7) The basic design assumptions (a two-hinged tied frame with hinges restrained by the tie slab against horizontal movement, but free to rotate, and a sufficient flexibility in the subpiers to warrant the neglect of their restraining action) were correct.

(8) The very close agreement between stresses measured by the cut-bar technique, those measured by the usual extensometer procedure, and those computed theoretically (obtained under widely varying temperatures) leads to the conclusion that temperature stresses, shrinkage stresses, and stresses originating in backfill effort and subpier restraint are quite small. Conclusion (8) is further indicated by the observed action of the abutment hinges under temperature change.

Under dead load the fascia frame is stressed in close agreement with the theoretical. It was not possible to use the cut-bar technique in the measurement of fascia girder stresses, so that no measured value was secured of the stress combination present under dead load; it was also impossible to secure stress-measurement data on an entire fascia frame, since only one half of the east fascia frame was observed. Accordingly, the objection was limited to a determination of stress under dead load by standard procedure, and to a comparison with the theoretical values used in proportioning. Significant quantitative stress-measurement data, from which to make general conclusions concerning the elastic effect of skew, were not secured in this phase of the investigation.

Stress Measurement Under Test Live Load I.—Test Live Load I was a test loading of 65 tons of cement in sacks distributed uniformly and transversely along the crown key on the skew parallel to abutments. This loading produced, effectively, a load concentration of 5.5 tons at the crown or midspan point of each deck girder. Extensometer stress measurements showed the amount of this loading to be insufficient to produce deformations of any significance. and the loading was increased, therefore, to that of Test Live Load II.

Stress Measurements Under Test Live Load II.—Test Live Load II was a test loading of 105 tons of cement in sacks distributed uniformly and transversely along the crown key on the skew parallel to abutments. It produced, effectively, a load concentration of 8.1 tons at the crown or midspan point of each deck girder. By placing this load on the skew, the effect of skew was probably eliminated entirely from the measured stresses of both the interior and the fascia frame.

Extensometer measurements showed that this loading was likewise insufficient in amount to produce deformations of significance, and the loading was therefore increased to that of Test Live Load IV.

Stress Measurements Under Test Live Load III.—Test Live Load III was a test loading of 78.7 tons of cement in sacks placed directly on the center-line interior girder between abutment lines. It very closely simulates the live load of conventional design, composed of two 24-ton trucks surrounded by a uniform live load of 100 lb per sq ft. This test loading was applied in such a manner that the entire weight rested upon the girder proper and no weight rested upon the flange slab. Thus placed, the true lateral distributive value of the flange slab is more easily studied. The characteristics of Test Live Load III are shown in Fig. 9.

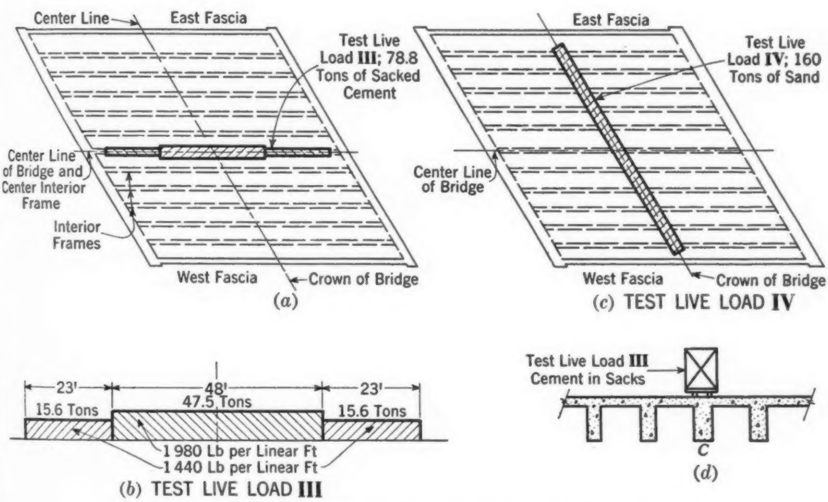

FIG. 9.—CHARACTERISTICS OF TEST LIVE LOADS III AND IV

Stress measurements on the interior frame under Test Live Load III are summarized in Table 1(*b*). A graphical comparison of computed and measured stresses and moments is shown in Figs. 7(*b*) and 8(*b*). The following conclusions seem warranted by the data secured in the stress measurements under this load:

(9) The structure functions as an elastic two-hinged tied frame with true measured stress functions very close to those indicated by the elastic theory.

(10) The measured crown moment, which was 71% of the theoretical value, indicates that there is apparently some lateral distribution of the load to the adjacent unloaded deck girders via the deck-slab flange.

(11) The measured haunch moments, which average 50% of the value obtained by conventional assumptions as to load distribution, indicate that

there is evidently an appreciable lateral distribution to the adjacent unloaded deck girders via the deck-slab flange and the cellular haunch slabs.

Stress measurements on the fascia frame under Test Live Load III indicate the existence of what is probably a complex torsional condition resulting from the skew of the structure. The objectives of this investigation did not embrace a study of this phase in the fascia and outer frames. The results secured by the stress measurements on the fascia frame were satisfactory and indicate that the stress under Test Live Load III is within safe limits.

Stress Measurements Under Test Live Load IV.—Test Live Load IV (Table 1(*c*)) was a test loading of 159.7 tons of bank sand retained in a timber crib, distributed uniformly and transversely along the crown key on the skew parallel to abutments. It produced, effectively, a load concentration of 13.8 tons at the crown or midspan point of each deck girder. Stress measurements were limited to the center-line girder only. This loading was 1.7 times heavier than Test Live Load II, and the stress measurements were made under a much smaller temperature range. The magnitude of the temperature corrections applied to the station measurements was considerably smaller in each case and resulted in a much smaller error from this source.

In this series the actual measured stresses were found to be in close agreement with the theoretical; in fact, the average variation was only about 12%. A comparison of the theoretical bending moments and the measured resisting moments, computed from the measured tensile stresses and the theoretical locations of the centroidal axis, indicates that under Test Live Load IV the following conclusions are warranted:

(12) The interior frames are stressed close to the theoretical values computed by means of the elastic theory.

(13) The basic design assumptions (a two-hinged tied frame with hinges restrained by the tie-slab against horizontal movement, but free to rotate, and a sufficient flexibility in the subpiers to warrant the neglect of their restraining action) were correct.

(14) The close agreement of measured and theoretical stress functions indicates that subpier restraint and backfill effort are quite negligible in modifying appreciably the stress condition of the superstructure.

Service Load Test.—It was considered desirable to make a service load test upon the completed bridge for assurance of safety under severe conditions. To effect this, the loading of Test Live Load IV was increased to 180 tons and an additional 60 tons of loaded trucks and tractors were arranged along the crown in midspan, bringing the total loading to 240 tons. To augment this static load, and to impose a still more severe condition, a 15-ton tractor was maneuvered rapidly about the deck in a manner to produce heavy impact shocks. Although this service load did not even begin to approximate the capacity of the structure, it was a severe load and is probably the largest the structure as a whole will ever be called upon to accommodate.

Conclusions

In addition to those presented in the body of the paper, the following general conclusions are offered:

(15) The basic structural assumptions that were made in the original design analysis and in the detailed proportioning of the components of the bridge were found to be valid and correct. They were well sustained by each phase of the investigation.

(16) The effect of active or passive backfill pressures is negligible in modifying appreciably the stress condition of the superstructure. Likewise negligible is the effect of the restraint developed by the subpiers upon the stress condition of the superstructure.

(17) The interior frames function, structurally, very closely as predicted by the elastic theory. In these frames the effect of skew is apparently very small for skewed symmetrical loadings similar to Test Live Load IV, and it may be neglected in proportioning the components in design. Under Test Live Load III there was a lateral distribution of load to the adjacent unloaded frames via the flange slabs. Under dead load and under Test Live Load IV the interior frames are stressed very closely to the theoretical values.

(18) The fascia frames under dead load function similarly to the interior frames. Under a symmetrical loading they show the effect of skew by a departure from the pattern common to the interior frames. However, proportioning the fascia frames by conventional methods is commensurate with safety.

(19) The assumption made in design that the hinges move freely in response to temperature variations in the superstructure, relatively unimpeded by the backfill and subpier resistance, was found to be valid. This assumption is particularly sustained by the study made of the hinge movements where the day-to-day variation in true hinge distance was found to be caused entirely by changes in length of the deck under temperature variations.

(20) By means of the cut-bar technique, the true, total, unmodified stress of all origins existing in the reinforcing at the time of measurement under the dead-load condition may be determined accurately. Use of this technique furnishes a close check on the measurements of dead-load stress secured by the standard procedure.

Acknowledgment

The original thesis from which this paper was prepared was presented to the Armour Institute of Technology (later part of the Illinois Institute of Technology), in Chicago, in 1940, in partial fulfilment of the requirements for the professional degree of Civil Engineer. The extensometer used was devised by Prof. Phillip C. Huntly of Armour College of Engineering, Illinois Institute of Technology, Chicago. It was made in the Institute shop under Prof. Huntly's direction. The instrument is not patented. In addition to Professor Huntly, the writers wish to acknowledge, especially, the valuable advice given them in this investigation by C. Thomas Kelly, and Edward Smulski.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

EVALUATION OF FLOOD LOSSES AND BENEFITS

By Edgar E. Foster,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The analysis of the damages, including the annual flood loss, constitutes the outstanding problem of the economics of flood control in which benefits must be weighed against costs. It is the purpose of this paper to present some methods that have been used by the writer in the U. S. Engineer Department. The primary condition imposed is that the method must be based on sound principles of mathematics, hydrology, and economics. In order to demonstrate that this condition was met, the paper contains a brief description of the various types of damage, some fundamental concepts of economics, a discussion of frequency and damage curves, and an example of the computation of the annual loss.

Introduction

Economic studies form an essential part of the investigation of practically every constructive enterprise, and works for flood control or flood protection are no exception, particularly when the loss of human life is not a factor. Indeed, lacking (as they usually do) any direct esthetic or intellectual appeal, their justification must be made primarily upon an economic basis. Although it is true that in great catastrophic floods the loss of life is appalling and the prevention of this loss is also a justification, nevertheless the costs of flood control are necessarily measured in economic terms and, to make adequate comparison of cost against benefit, the latter must be gaged on the same basis. Moreover, aside from the loss of life, flood control or protection is entirely an economic matter.

The losses from which the benefits are computed may be found by investigation or appraisal, and the costs can be estimated in the same manner as costs of other structures. When the losses are known, the annual benefits can be computed and compared with annual costs. This is the object of the

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economic study. On this comparison depends: (1) Whether or not any protective measures are justified; (2) to what extent protection may be given; and (3) in part, what kind of works may be provided.

The analysis of flood losses and computation of annual loss must proceed on a rational basis as far as such bases can be found in the relevant fields of economics, mathematics, and hydrology. It is the purpose of this paper to develop some methods applicable to such analysis.

Description of Flood Damage

Flood losses arise from the occupation of the fluvial plains that are sufficiently high to escape the frequent and smaller freshets but are still low enough to be inundated by the greater and rarer floods. From the beginning of civilization, the flood plains have been favored lands for settlement. Prior to the development of modern railways, rivers formed the most important means of commercial transportation and commerce, which required a settlement where navigation could be had if such a site were at all possible. This factor is probably the most potent of all that promoted riverside settlement and, moreover, is a world-wide factor in city growth. Another reason for such settlement is the ease of construction on the flat terrane. This means cheaper construction for other and more modern forms of transportation, as railways and highways. In some cases, notably in New England, the existence of potential water power induced the development of industries on the banks of rivers. Then, regardless of the original reason for settlement, once a city is established on a flood plain, it will stay. People are so loath to leave their property that only the worst catastrophes will force them to leave any except the smallest towns. Therefore, as long as cities and towns occupy flood plains, and as long as rivers have floods, there will be flood losses from which protection will be sought.

The diversity of losses is as great as the economic interests of society itself, since floods inundate everything on the plain as, perhaps, may be the right of floods. However, since society has found the land of flood plains most desirable for nearly every human purpose, it will continue to occupy that land and consequently continue to suffer losses in every line of activity. Industrial, commercial, utility, municipal, recreational, social, and agricultural interests all pay toll for trespassing on a flood plain, and others, throughout the present-day closely knit social and economic organization, contribute in a diminishing degree, although far from the scene of disaster.

Industrial losses include those sustained by establishments engaged in manufacturing activities in which raw materials are processed to form a finished product for use by a consumer or in another manufactory. Usually the structures used for manufacturing purposes are sufficiently strong to withstand the onslaught of a flood and will survive unless they are located so close to the riverbank that they become undermined. Small auxiliary buildings may be washed away or wrecked by other means. Wooden floors will swell and bulge so that extensive repairs or replacements are required. Floor covering and wall finishing will be damaged and windows will be broken. Machinery is injured to a varying degree that depends upon the type of machine. Engines and pumps and similar heavy machines will not suffer any great damage from simple immersion but will require dismantling and thorough cleansing. Electrical machinery, particularly motors and generators, requires thorough cleansing and even more thorough and careful drying. Even then, such machinery is subject to rapid deterioration and early replacement. The more fragile machinery, such as is used in the textile industry, is subject to breakage by floating debris as well as the foregoing damage. Many classes of raw materials and finished products, such as textiles and clothing, are liable to total destruction. Others (some heavy rubber goods, for example) may be reclaimed by thorough cleaning. Still others may be damaged in quality so that their value is reduced. Some industries (rayon, for example) that operate in a continuous process suffer a loss simply because of the untimely interruption which necessitates clearing the plant of material in process.

Industrial losses also arise from the destruction or damage to records if the administrative offices are so located that they too are flooded. Some papers may be recovered by careful drying but others may be a total loss. A case of the latter kind occurred in the Merrimack River Valley in the flood of March, 1936. The owner of a chemical manufacturing plant had some chemical formulas that had been obtained after several years of study and experimentation. The formulas, having been written out and left in the office, were completely destroyed by the flood and could be reproduced only by repeating the investigations at a heavy cost. Furthermore, all factories suffer heavy indirect losses because of the interruption of the usual business with a continuation of fixed costs. This loss may be incurred whether or not the plants are located in the flood zone, but, of course, not to the same extent.

Under commercial losses may be included all those sustained by establishments engaged in trade, wholesale and retail; banks and other financial institutions; professional and other offices; service concerns such as laundries, plumbers, barbers, beauty shops, restaurants and hotels, theaters, and similar lines of business that are too numerous and diversified to list in detail.

Retail and wholesale shops lose heavily in merchandise that has been submerged. Goods that have been soaked by dirty and polluted flood water are usually a total loss, although there may be some kinds that may be reclaimed to sell at much reduced prices. Damage to accounts and records results in a loss, the minimum value of which is the cost of restoring them by cleansing and drying into some recognizable condition. Buildings and fixtures are always damaged at least to the extent of requiring cleaning and redecorating. Plate-glass fronts, which are a conspicuous feature of retail shops, are heavy sufferers if the flood rises high enough to carry debris against them. Likewise, all affected concerns sustain an indirect loss because of suspension of business.

Under the term "utilities" may be grouped all agencies or private and public organizations that provide or sell some form of service to the consumer or patron. These agencies include electric, gas, telephone and telegraph, and railroad companies, and water works, among private organizations; under public ownership may be included water works, sewerage systems, and streets and highways.

Since utilities deal in service rather than goods, there is relatively little to be lost in stocks of goods, either in raw materials or finished products of their principal business. Gas companies, however, may lose gas from their containers or pipe lines, and both gas and electric utilities may suffer damage to merchandise which is sold for the utilization of their services. Both also may lose stocks of operating supplies, and both may sustain losses in business due to the inundation of their patrons and the suspension of other business. Hydro-electric power houses and dams and river crossings of pipe lines and transmission lines are danger points and frequently are damaged severely.

Railroads and highways suffer because grades are damaged by slides or are washed out entirely, taking pavements or rails with them as the case may be. Railroads sustain damage to rolling stock, which is lost not infrequently when freight cars are loaded with rock or other ballast and placed on bridges that are in danger of going out. Both railroads and highways suffer loss of bridges. In many cases the entire bridge is destroyed; in others, the piers may be damaged or the approaches eroded. Occasionally the bridge superstructure may be damaged, although left in place. The submergence of both highways and railways causes a temporary rerouting of traffic with consequent greater cost to the public in the case of the former and to the railway company in the latter case.

Dams, like bridges and all other structures in rivers, are particularly vulnerable to floods. Flashboards may be washed away by any freshet, gates may be damaged, or the abutments washed out by heavy and high water; or the entire dam may fail and release the water in storage to augment the flood below. Because a dam has successfully passed one flood is no reason to be certain that it will pass all others. All structures in the channels are subject to damage and may be weakened by successive floods until one finally washes them out.

Under the term "municipal" losses may be grouped all damage to property and all increased cost of municipal operation during the flood: These include all losses sustained by city, county, state, or other governmental agencies, including the federal government.

Streets and roads suffer heavy damage in flood from washing and erosion, and heavy deposits of debris must be removed after the water has subsided. Sewer outlets in streams and water-main crossings are subject to damage. Municipal buildings are liable to the same kind of damage as other structures, because floods are no respecters of authority. In addition to the physical loss, the government must spend large sums of money for relief of those who are driven from their homes. The police duties are increased by the emergency, and frequently other forces are required to maintain civic order, the cost of which is a true flood loss.

Farm and rural losses include all damage sustained by agricultural lands and forests, together with all appurtenant improvements. This includes the loss of livestock that may be trapped and drowned; damage or destruction of buildings and farm equipment; loss of crops that may be in the field, growing or harvested; and loss due to prevention of planting, erosion of valuable land, or deposits of debris that may render the soil untillable. The flood may wash out fertilizer that has been placed on the land in preparation for crop, or it may necessitate repeating the work of preparing the field for planting. It may erode the topsoil so that replacement may be necessary or one or more years of fruitless cultivation may be required to restore the fertility.

Residential losses include all damage to dwellings, both the buildings and the grounds, and the furnishings and other personal property commonly used or kept around homes. As in other types of damage, the extent varies greatly as the desires and means of the occupant vary. Entire houses may be carried away if they are located in areas submerged by swift, deep water or they may be undermined by erosion of the banks of the stream. Buildings, household goods, and generally the land may be charged off as a total loss in cases where the entire house is taken away. In other cases the personal property is damaged by immersion; furniture and other household furnishings are badly damaged and many pieces (pianos, for example) must be replaced. The dwelling must be thoroughly cleansed and redecorated and may require more or less extensive repair work. Moreover, there is usually some cost of moving out because of rising water and providing other habitation for the duration of the flood. This cost is properly chargeable to flood losses.

The term "social losses" may apply to all losses arising from the interruption of normal, individual, or collective social activity and to the personal risk of life and health endangered by floods. The term is necessarily rather vague and very broad. The losses are indefinite in most cases and may be practically nothing, or they may be large. For example, an interruption of an afternoon bridge game is an inconvenience but the loss caused by such an interruption is negligible or nonexistent. On the other hand, the personal risk may be very real. The inundation may result in sickness and death, a loss that cannot be measured in terms of money. These social losses may be considered as indirect losses accompanying the direct residential damage.

Some Definitions of Wealth and Value

The considerations of flood damages and benefits involve the most basic and fundamental concepts of the science of economics. It is desirable to define these concepts herein before proceeding to the analysis of losses.

The first concept is that of wealth. It is necessary to know what can be classed logically as the real wealth of a community that has suffered flood damage in order to determine the true loss. By the term "wealth" is meant the sum of all goods, fixed or movable, which have value to individuals or organizations of individuals who have a commonly recognized title to ownership of the goods or property in question. Wealth may be owned by private or public agencies. "Wealth can be described by means of a stock-taking or inventory, but it can be summed or measured as a whole only in terms of its money value. * * * Its value is the sum of the values of existing property rights."[2]

Economic goods may be divided into two general classes—productive and consumptive goods. These two classes include all forms of property that constitute the real wealth of the community or the country. They include

[2] "Wealth," *The Encyclopædia Britannica*, 14th Ed., Vol. 23, p. 448.

both real and personal property. They do not, however, include evidences of indebtedness, such as bonds, because such papers are essentially claims to other property, the title of which rests in other owners.

Productive property is taken to be that which is devoted to the use of producing other goods or service in commercial operations. It includes both real and personal property—that is, land and improvements and all equipment devoted to such production. It consists of the property used in all commerce, industry, utilities, and agriculture; it is fundamentally all property used to produce an income.

Consumptive or nonproductive property, on the other hand, includes all property, both real and personal, put to uses that do not produce other goods or commercial services. Nonproductive fixed property is essentially a kind of "consumptive good" because it is used directly by the consumer or patron. Its utility is obtained directly by the user. Nonproductive property includes residences, religious and educational institutions (both the land and buildings), libraries and art galleries, parks and similar recreational areas for which admission is not charged, streets, highways, sewers (generally), and governmental and municipal structures. This differentiation between productive and consumptive property is essential to the proper comprehension of the effect of flood losses on depreciation.

Another concept for which a definition must be made is value. For the purposes of this paper, two definitions are given, interrelated but viewing the concept differently. First, the value of any particular item of property may be defined or measured as the quantity of other goods that may be given or received in exchange; under existing economic conditions the "other goods" is nearly always money. The value arrived at will depend upon two or more parties involved in a transaction. This is the exchange or objective value and is the definition usually in mind when the worth of goods is discussed.

In contrast with the exchange value is the subjective value, which is the worth of an item of property in the consideration of the holder in comparison with other goods. This value depends upon an individual's sentiments and preferences, without effective regard to the utility of the property in question. It cannot be measured directly in money since the holder usually places a greater consideration on the property than its utility to others is worth.

Actually, the subjective value enters the appraisal of a personal need and the purchase of every article of consumptive goods obtained for the usefulness to the purchaser. The particular item that is bought will be what the buyer believes will give him greater utility than some other "good" that may be had for the same money. This displacement value of consumptive goods is entirely subjective, but in the case of production goods it will depend upon the relative costs of production attainable and may be measured or estimated in terms of money. An item, therefore, is worth to the consumer what he will pay for it and to the producer what he can earn with it.

If the item of property is offered for sale at a price within the replacement value, the objective or exchange value is reached. On the other hand, if the individual refuses to pay the exchange price or cost, it should be safely inferred that the subjective value is not worth to him as much as is asked. Questions of indirect losses, business losses, depreciation of property, risk of health and life, and evaluation of indirect benefits must be answered by reference to the basic concepts of value.

Definition of Flood Losses and Benefits

It becomes necessary now to make some definitions of what constitute valid flood losses and benefits. The economic effect of floods touches such a multitude of diverse phases of human activity that, unless some guiding principles are established for analysis, confusion and, consequently, erroneous conclusions will be deduced. That these principles must be basically sound and in accordance with well-established economic doctrines that are recognized to be fundamentally correct goes as a matter of course. Because of the aforementioned diversity of flood losses, the principles and definitions can be only the broadest.

Flood losses may be defined as those losses arising from the following exigencies:

(*a*) *Any Destruction or Damage Caused by a Flood to the Existing Real or Personal Property of a Community.*—This includes all physical and such intangible property as may constitute a part of the total wealth of the community. As a rule, it will not include the loss of the evidence of indebtedness; only where the debtor is outside the damaged community and when the debt cannot be proved except by the papers lost can the value of the debt be allowed as a valid loss, and in that case it is a loss only in that community and not in an area including both the debtor and the creditor.

The operation of this rule can be shown clearly by an illustrative property account, which may be called the Community Property Account (See Fig. 1).

Community Property Account

(Debits) (Credits)

Date		Item			Amount	Date		Item			Amount
		Property of *A* Property of *B* Property of *C*			X_1 X_2 X_3			Community wealth			X_n
		Total			X_n			Total			X_n

Fig. 1

The account must balance. Hence, any loss that affects the property of *A* (who, it is assumed, has no connection with the others) will result in a decrease of the debits and necessarily will cause a reduction on the credit side of the account, which represents the total community wealth. Suppose again that *B* sustains a loss which is a payment that goes entirely to *C*. There is no change in the debits, and consequently there will be no change in the community wealth.

(*b*) *Loss of Earnings or Loss of Services.*—Earnings, whether they are wages, salaries, or professional fees, are compensation for services and are an economic commodity, as are material goods. It follows, therefore, that loss of services constitutes an economic loss to the community, but that both loss of services and loss of compensation should not be counted is brought out by considering a Community Income Account similar to the Community Property Account in Fig. 1. Assume that D works for E and that E's plant is closed because of the flood. In this case E loses the services but saves the payment of the wages, and D loses his compensation; there are two individual losses and one individual gain, which leaves one net loss to the community—the loss of D's services, which have the value of the compensation. This rule will be applied later in the consideration of business losses.

(*c*) *Increased Expense of Usual Operations.*—Floods interfere with normal operations and cause increased expense either without, or in addition to, other damage. This situation is likely to be encountered in the utility field, which is obliged to maintain service at all times regardless of the emergency. Certain functions of the municipal governments are likewise greatly increased in periods of flood. A somewhat similar type of loss exists in all commercial or industrial firms. They have certain overhead expenses, such as taxes, rent, insurance, and so forth, that must be paid regardless of interruption of income. When the interruption is a flood, the expenditures for fixed charges or overhead may be fairly charged to flood losses.

(*d*) *Diminution of Capital Value of Real Estate.*—This type of loss is emphasized in the case of residential property and is applicable to nearly all nonproductive or consumptive property. The repairs of flood damage, which constitute an added expense, and the personal risk involved, make this property less desirable for use. The subjective evaluation of this lessened desirability is reflected in the capital value by diminishing it. Productive property is subject to the same capital loss as a result of a flood not previously experienced or expected. If such property is located on a site known to be subject to floods, that added expense should be foreseen and the capital value adjusted or fixed on the basis of such anticipated expense, which becomes a charge against income. For all property, however, this diminution of property value should be considered as a nonrecurring loss if it is allowed in one flood. This type of loss will be considered again in connection with indirect losses and one method of evaluation of flood damages.

(*e*) *Expenditures Caused by the Flood Emergency or a Threat of Flood.*—These expenditures may be roughly grouped into two classes: (1) Emergency flood protection, and (2) cost of relief to flood sufferers.

Whenever a sufficient warning is had, occupants of the flood plain take steps to protect their property. Dikes may be constructed to protect low areas; machinery may be moved to high ground or to upper stories or may be protected by other means; and stocks of merchandise are raised to elevations above the expected flood. This work is undertaken before it can be known to what stage the water will rise, and entails expense—frequently heavy expense—which must be met from some source. These expenditures are properly chargeable as flood losses even though it is proved later to have been needless.

Floods invariably drive large numbers of people from their homes and cause the loss of their usual source of subsistence and shelter, which must be furnished from other sources. The refugees gain what is expended by the municipality and relief agencies, but the loss of their usual sources of subsistence and shelter is probably worth considerably more to them on account of the subjective value, if for no other reason. The full amount of relief expenditures may reasonably be taken as a flood loss.

Two General Classes of Flood Losses

For the purpose of evaluating annual flood losses and benefits, it is convenient to divide the damages into two general classes—direct and indirect. The broad basis of division is the immediate origin of the damage; but since the object of the classification is convenience rather than inescapable necessity, other factors may be given consideration that may entail variations in treatment. In the category of direct losses may be placed all damages to physical property (Class (*a*)), and the direct expenditures (Class (*e*)) for flood emergency.

In addition to the direct losses sustained by the physical property, other losses (Classes (*b*) and (*c*)) arise indirectly from many sources because of the flood. These constitute the "indirect losses." The sources of the indirect losses are as diverse as the direct. They arise in the flood zone and outside, and may occur far from the locality of the flood.

Perhaps first in importance among the sources of indirect flood losses is the interruption of commercial activity. In this respect floods are similar to fires; they stop the normal earning capacity of all commerce and industry for the duration of the high water and for whatever length of time is needed to clean the premises and repair the damage. This business loss consists of the gross business profit of the operations. It includes loss of profit, wages and salaries, all expense for overhead, and all indirect operating costs. Where no goods are manufactured or sold, no raw material or stock of goods will be used, and consequently the difference between the cost of raw materials or cost of stock of merchandise and the normal value of the goods sold will form the indirect flood loss caused by the interruption of business.

The data obtainable for computing this kind of indirect loss consist of the value of sales for the period of suspended activity, and they should be examined critically. The amount of sales should be normal for the season. Items of cost that are suspended or reduced should be omitted from the losses. It commonly happens that, after the flood has subsided, labor normally used in the business is assigned to cleaning and repairing the plant. In this case, labor costs should be charged to direct loss and not the indirect loss. For computing the indirect loss from the normal sales, rates of gross profit, less labor costs (usually), may be estimated for each line of business and applied to the data of loss of sales.

Another source of indirect loss arises in the transportation systems—the railroads and the highways. When the highways and railroad tracks are submerged, the grades eroded, or bridges damaged or destroyed, the traffic must use other, and in practically all cases, longer routes. This necessarily results in greater expense of transportation. In the case of the railroads, the cost is borne by the companies and could probably be estimated from their records, although a search would be long and tedious. Furthermore, there is likely to be some loss or at least inconvenience to the consignees or shippers and the passengers because of the longer route.

In contrast with rail traffic, the losses caused by floods to highway transportation are borne by the public. The individual losses may be little or they may be considerable, for they will depend upon how much use is made of the roads by the individual. With rail traffic, these losses arise because of rerouting and consequent longer haul. For example, it may cost 1.5 cents per mile to operate an ordinary passenger car. If, then, that car is forced to use a route 5 miles longer, and is forced to traverse it four times daily, it will sustain an additional expense of 30 cents a day for the duration of the interruption. One or several cars will not produce a great indirect loss; but the operation of many vehicles, including commercial trucks with much higher operating cost, can amount to a considerable sum in a short time.

The interruption of the services of utilities can cause the suspension of operations of business outside of the flood zone. Electric power is particularly vulnerable in this respect, because hydroelectric plants (which usually generate a part of the supply) and transmission lines at river crossings form prospective sources of interruptions.

There are other sources of indirect losses, most of which are even more difficult to express in terms of money. There is the loss of the use of playgrounds, parks, schools, churches, and similar property. Each flood brings with it a definite risk of loss of health and frequently life that can be evaluated only subjectively. No attempt can be made herein to enumerate all possible forms of indirect losses, because there are manifold sources, but the foregoing are the principal ones. Others should be searched for in each flood.

Flood Risk

Before proceeding to describe the methods of evaluating the annual flood loss, it is desirable to discuss further the economic risk of such damage. Although floods are usually more likely to occur in some seasons than in others, the time of occurrence is unknown until the rising stages may give warning of impending danger. Even that foreknowledge is available only on the larger rivers, and disastrous floods may occur on smaller streams without any indications of their approach. The magnitude, perhaps, is more uncertain than the time of occurrence. There are many areas, moreover, which may be inundated but yet have not in recorded history experienced a freshet great enough to cover them. The occupants are subject to flood damage, therefore, even though it is unknown to themselves. These people who dwell or work on a flood plain are thus liable to an unknown and uncertain expense that can be designated the "flood risk." Economically, this flood risk is in no way different from that of other uncertain catastrophic events such as fire or earthquake.

From an economic standpoint,

> "* * * risk may be defined as uncertainty in regard to cost, loss, or damage. In this definition, emphasis is on the word 'uncertainty.' Where destruction of capital is certain in connection with a business process, it can be charged up in advance as a cost. It is not a risk. When the destruction or loss is uncertain, it may be dealt with in accordance with the judgment of probabilities and presents a problem of risk."[3]

[3] "Risk and Risk-Bearing," by Charles O. Hardy, Inst. of Economics, Washington, D. C.

This definition fits the flood situation perfectly. Moreover, the definition may be extended a step further by defining the annual flood loss as the cost of the flood risk.

Flood risk, as well as other kinds, may be conveniently divided into two classes—property or capital, and personal risks. The first kind pertains to the loss of property or goods. It may be satisfied with the money value of goods or damage sustained. The personal risk is due to the subjective value that an individual puts on an object. It is usually greater than the exchange value. It is a value that arises from long association with the object in question, such as, for example, an old chair, store, house, or the building of a life business. The same category will cover the loss of means, employment, health, and life itself.

The capital loss can be evaluated satisfactorily on an annual basis when the property damage is known; but the personal risk can be evaluated only indirectly, either by determining the value from the adverse effects of the risk or by determining what the individual will pay to ameliorate his situation. This paper is concerned primarily with the property risk.

Since flood risk depends upon uncertain events, its evaluation must be based upon some expression of the probabilities of occurrence. These estimates of probability fall into three classes,[3] namely: (1) Mathematical *a priori*, (2) statistical, and (3) judgment. In a further statement, Charles O. Hardy states that judgment is only a crude application of the statistical method. In determining the probability of flooding, unaided judgment should be used only when there are no other means possible.

On the other hand, mathematical *a priori* methods cannot be used to determine the probability of floods, and hence that method is eliminated from further consideration. There remains, therefore, only the methods of statistics, which can be based on the records of flood data that have been compiled. It is on the principles of statistics that the methods of evaluating the annual flood losses, discussed in this paper, are based.

The Computation of Annual Flood Losses

The annual flood loss has been defined as the cost of the flood risk, and it may be considered as an expense to be met in the same manner as any other similar cost, such as fire losses. The cost is not uniform in the absence of insurance but varies as time and chance provide the floods. The risk of floods is continuous, however, the same as that of any other catastrophe; when it occurs the damage is done and the expense must be met.

The intermittent and sometimes heavy cost of floods may be reduced to an annual basis in accordance with the mathematical expectancy as expressed by the formula

$$A = p\,D \qquad (1)$$

in which A is the annual flood loss, p the probable number of floods per annum, and D the total damage caused by the floods. Eq. 1 is well known and is the mathematical expectancy of payment or cost of a chance event. The number of floods per annum can be taken from a frequency curve which must be computed from a record of flood discharge. The damage, D, must be taken from a stage-damage or similar curve that has been constructed from data obtained from a field survey.

Frequency Curves

It can be seen from Eq. 1 that the probability of a flood is one of the two essential factors entering into the computation of the expected annual flood loss. Therefore, it is advisable that some consideration be given to this matter. Although data of floods will be approximations at best, they should nevertheless be analyzed carefully, and the frequency should be computed by the best possible means.

Many methods of computing flood frequencies have been proposed, some purely empirical and others partly based on the mathematical theory of probability. It is plain, however, that the importance of accurate evaluation of the annual flood losses is such that the frequencies should be based on a sound mathematical and hydrological theory. Although it is not the purpose of this paper to enter into a long discussion of the various methods of computing frequencies of floods, it seems, nevertheless, desirable to describe briefly the applicability of the theoretical methods.

Floods are the result of extraordinary rainfall, or rapid melting of snow, or a combination of both. Behind each of these two primary causes there must function a long chain of preliminary events, each of which has its own peculiar variations and may operate in various degrees in the combination of circumstances that produce the flood. In fact, a peak is a result of fortuitous combination of favorable chance variations of the usual meteorological events. For that reason, data of floods, both of magnitude and of number, are amenable to treatment by the mathematical theory of probability.

Since the magnitude and the number of floods are the result of chance circumstances, one cannot expect a given series to be duplicated. The engineer must anticipate that there will be floods in the future as in the past; but when, or how big, a given flood will be he does not know; and he has no way of learning. On the other hand, he may expect a similar distribution of floods (that is, a similar proportion of high, moderate, and low peaks in a given period), and from the distribution he may expect approximately the same average and the same mean square variation (or coefficient of variation). These two quantities constitute the first and second statistical moments. If the record is long enough, it is also reasonable to anticipate similarity in the higher statistical moments, and from a record of moderate length the moment may be obtained with suitable accuracy. The frequency of floods then may be computed from the moments by the mathematical theory of probability.

Several sound methods exist that may be used for the computation of frequency curves. First in point of publication is the method proposed by H. Alden Foster,[4] M. Am. Soc. C. E., for which the probability functions of Karl Pearson formed the basis. Secondly, J. J. Slade, Jr.,[5] presented an asymmetrical probability function that can be used appropriately for computing

[4] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

[5] *Loc. cit.*, Vol. 101 (1936), p. 35.

frequencies of peak discharge or volume. If the data of floods consist of stages only, a frequency curve can be computed readily by means of a Gram-Charlier probability series.[6] In using the same data these functions will yield closely comparable results. They agree very closely for the range of data which the former will cover. Likewise Professor Slade's function gives results practically identical with those obtained by means of a Gram-Charlier series when the data consist of stages.

Before attempting to construct any frequency curve, the records of discharge should be examined carefully to determine whether or not the data of crests, especially in the earlier part of the period, are comparable to those of current observation. The regimen of streams, even in the natural state, changes more or less as time passes. The greatest danger is that conditions affecting stream flow may have been altered by human agencies in the development along the bank in a way that changes the stage-discharge relationship or even modifies the discharge. In such an examination, each record must be treated on its merits and given individual attention. Nevertheless, there are some sources of modification for which a special search should be made. Some of these factors modify discharge and the resulting stage, and others change the stage-discharge relationship so that only the stage is affected.

First, there may be diversion from one watershed to another. This diversion is most likely to occur in projects for municipal water supply but may arise in development for irrigation or water power. All discharges will be reduced, but the low-water flow will be affected most appreciably. If the diversion works provide a spillway into the old stream, the effect on floods may be relatively small or even negligible.

Second, there may be storage reservoirs upstream from the gaging station. These reservoirs will modify all floods to an extent that will depend upon the capacity of the reservoir and the drainage area above. Small reservoirs well up in the headwaters and controlling only a small part of the watershed may be neglected. The effect of the larger ones, especially those closely above the gaging point, must be considered. The best procedure, perhaps, is to compute the probable effect the reservoir would have had on the crests that occurred previous to its construction. At best, however, a new uncertainty has been introduced in the data by the vagaries of the human operation of the reservoir.

Third, the construction of levees for local flood protection will modify greatly the stage-discharge relationship, although it will not affect the rate of discharge appreciably unless much valley storage is lost.

Fourth, encroachment on the channel by bridges, highway and railroad grades, and the construction of buildings on the flood plain and channel modify the crests chiefly by causing backwater which changes the stage-discharge relationship.

Fifth, the construction of navigation dams and run-of-river power plants that have little or no storage affects the crests to some extent. It is probable that works of this type will have little or no effect on floods downstream other than perhaps to shift the time of crest slightly. The effect on the

[6] "Probability and Its Engineering Uses," by Thomas C. Fry, D. Van Nostrand Co., Inc., N. Y., 1928.

reduction of flood crests of power dams with pondage only is likely to be overestimated, but it should be investigated.

Sixth, there is usually some intermittent erosion and shifting of the channel. This situation is likely to occur when the stream flows through a loose geological formation such as alluvial silt. The rate of flow will not be affected, but the stage-discharge relationship is very uncertain. Unless the entire record can be worked over with reliable discharge rating curves or the entire discharge can be based on frequent—almost daily—measurements, it is better to compute a frequency curve of stage only.

Seventh, there are some other events that must be looked for, such as ice jams and dam failures. The effect of the former should be included in frequency curves to be used with damages possible during the spring or winter seasons. The latter are so rare that the effect should be eliminated from the flood record.

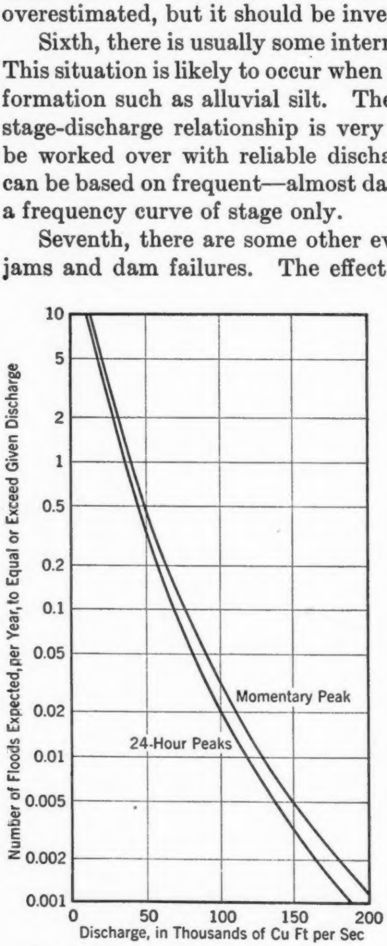

FIG. 2.—FLOOD FREQUENCY; MERRIMACK RIVER, LAWRENCE, MASS.

The preferable form of frequency curve is obtained by plotting the number of peaks or floods expected per year against magnitude, either of discharge or stage. Quantities needed for the computation of the annual loss may then be read directly from the curve. Fig. 2 shows a sample curve computed from the records of the Merrimack River at Lawrence, Mass.

Because the potential damage causable by floods is not uniformly distributed through the year, it is sometimes necessary to make frequency curves for floods experienced in different seasons. The most satisfactory manner of accomplishing this is to use the data of floods to compute separate curves for the different seasons. This may be done where a long record with ample data exists. For each given frequency the sum of the number of floods expected during each season or fraction of the year should equal the total number for the year.

It may be necessary to make seasonal frequency curves from data of those periods during which a single cause of floods, such as melting snow, operates exclusively of others. Such a condition prevails in regions where early spring floods arise from melting snow prior to a season when intense rainstorms can occur. In other sections, such as the southern parts of the United States where heavy rainstorms are the sole primary cause of floods, the annual frequency curves may be divided into seasonal curves on the basis of the yearly distribution of the peak data.

Stage-Damage Curves

A stage-damage curve represents the losses caused by floods at various stages. An example, for Lawrence, is shown in Fig. 3. It is assumed in this example, and in all similar curves, that the total direct loss caused by a flood is given at any stage, regardless of how, or from what data, the curve was constructed. It may be constructed from data obtained by a survey of actual damage caused by one high flood, or from the damage data of a number of floods that have occurred at different times and to diverse stages, or it may be computed from the estimated potential damage. In any case, a point on the curve should give the total direct loss from a flood to the corresponding stage.

A variant of the stage-damage curve is one which shows the loss plotted against peak discharge. The discharge-damage curve will frequently facilitate computation of the annual loss.

The stage-damage curve links the damage to the frequency of floods through the elevation of the property to the stages of the stream. The essential data of each item of damage required for its construction are the location and the elevation of the damage above a datum plane which can be correlated with stages of the stream. The elevation of damage above ground or below a visible high-water mark can be obtained by measurement. All variations of elevation, whether caused by difference in topography or by construction of the property, are taken into account by the stage-damage curve.

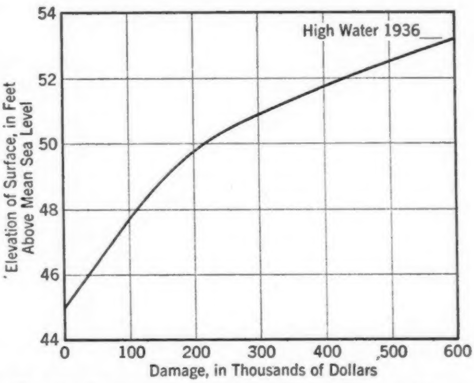

Fig. 3.—Stage-Damage Curve for Lawrence, Mass.

An example of the computation of the annual loss of an urban area is shown in Table 1(*a*). The frequencies are taken from Fig. 2 and the damage from Fig. 3. The stage, 53.1 ft (Table 1), is the maximum that has been experienced. The damage of greater floods has been limited to the same amount in this computation, as a matter of conservatism.

Estimating Future Flood Losses

Of course, data of flood damage can be obtained only for floods that have been experienced; there is no other basis on which data of actual, experienced losses can be had. Yet, since comparison must be made with future costs, it becomes necessary to use the data at hand to estimate future losses and benefits also. The compiled data should be examined carefully, therefore, to eliminate nonrecurring losses and sometimes to make adjustments for past economic growth or expected future trend of development.

Damage that manifestly will not occur again should be eliminated from the computation of annual losses. In this category may be included (1) bridges that have been washed out and will not be replaced, or new structures that have sufficient elevation or waterway capacity to obviate all future danger; (2) dams that will not be reconstructed after failure; (3) other buildings that have been destroyed and will not be replaced; and (4) areas that have been evacuated and cannot be reoccupied because of physical conditions or legal restrictions. Items of damage that are eliminated should be scrutinized carefully and cast out only where it is reasonably certain that other developments,

TABLE 1.—Annual Loss from Floods, Lawrence, Mass.

Stage, in ft	Discharge, in cu ft per sec	Frequency (No. per Yr)		(a) Natural Conditions			(b) Reduced 1 Ft		
		For discharge	For interval	Damage (Thousand Dollars): For discharge	Damage (Thousand Dollars): For interval	Annual loss (dollars)	Damage (Thousand Dollars): For discharge	Damage (Thousand Dollars): For interval	Annual loss (dollars)
45.1	47,700	0.530		0			0		
			0.300		20.5	6,150		0	0
46.1	62,000	0.230		41			0		
			0.130		59.5	7,735		20.5	2,665
47.1	77,000	0.100		78			41		
			0.053		96.0	5,078		59.5	3,153
48.1	92,100	0.047		114			68		
			0.025		136.0	3,400		96.0	2,400
49.1	108,000	0.0220		158			114		
			0.0100		192.5	1,925		136.0	1,360
50.1	124,900	0.0120		227			158		
			0.0056		275.0	1,540		192.5	1,078
51.1	141,700	0.0064		323			227		
			0.0027		384.5	1,038		275.0	742
52.1	158,700	0.0037		446			323		
			0.00125		518.0	648		384.5	481
53.1	174,000	0.00245		590			446		
			0.00245		590.0	1,446		446.0	1,093
Maximum Flood		0.00000		590			446		
Total annual loss		....	....	....	....	28,960	....	....	12,972

subject to similar damage, will not take their places. If replaced structures are vulnerable, the flood risk will remain the same as before the flood. Furthermore, unless some restrictions are made by conversion of land into parks or other legal enactment, evacuated areas may be reoccupied and the flood risk re-established. Moreover, there are unusual losses in nearly every flood so that an item should not be thrown out simply because it is unique.

It is impossible, of course, to predict definitely what economic development will take place in the future, but at least the trend must be examined in extending flood losses to compare with future costs. In this respect, flood-control works are on an equal footing with other structural developments, such as hydroelectric plants, railroads, large office buildings, and industrial plants. All must be constructed in the light of present economic conditions plus what trend may be predicted safely.

A key to economic development may be found in the growth of population, since all such activity arises from satisfying human needs. The growth of population in the United States apparently has become stabilized following the full settlement of the country, and relatively small changes may be expected in the future except in localities where a new industry has settled or some other attraction to development exists. These factors may be determined by investigation. Except in special circumstances, it will be in the interest of conservatism to assume no increasing losses in the future.

Some difficulty will be encountered in correcting or adjusting for seasonal changes in the potential flood damage. Usually the seasonal fluctuations of the retail trade are too small to be considered in view of the probable error of the flood-damage data. There are some industrial activities, however, which have a seasonal trend that may require some special consideration. Activities such as fairs, which commonly last only a week or ten days, will subject much perishable property to loss in event of a flood, if the grounds are in an area liable to inundation. Plainly, such potential loss cannot be computed with an annual frequency curve but should be used with the number of floods to be expected during the period of such activity.

Agricultural Losses

Other than crops, rural and agricultural losses can well be treated in the same manner as urban damage. The reaches may be longer and the losses will be more widely scattered, but otherwise there is no essential difference from urban damage, and there is no material seasonal fluctuation of risk. The outstanding source of such seasonal changes, of course, is agricultural crop losses, which must be given special consideration.

In contrast with urban and other rural damage, potential crop losses resulting from floods vary greatly from season to season; and they vary with the area flooded in a manner distinct from other damage. Beyond a relatively shallow depth, they do not increase with added inundation, for, if a crop is flooded sufficiently to be a total loss, it suffers no further damage by deeper water. These factors necessitate different treatment and entail the use of two other curves instead of the stage-damage curve. The first of these is the "area-flooded" curve, which represents the area of land flooded at various stages. The second is the "crop-loss" curve, which shows the potential loss of crops for the various seasons of the year.

The area-flooded curve may show the cultivated area inundated in a given reach or (preferably) the total area. The latter is the more constant value. The area flooded depends upon the stage and the topography of the valley or flood plain. Accurate topographic maps are necessary for its construction.

The risk of crop loss varies from nothing in the winter season in the northern latitudes to full, or nearly full, value of the harvested crop in summer. In southern or warm regions there may be some loss throughout the year because the work of preparation, such as plowing or the application of fertilizer, may be nullified by floods. The maximum monetary loss, of course, will depend also upon the value of the crop.

The "crop-loss" curve, which shows this seasonal flood risk, should preferably give the loss per acre, for clarity and convenience in computation. A curve may be used for each individual crop, or one curve may be a combination of several crops weighted by the average plantings of each. The example in Fig. 4 illustrates a curve of the latter type. Data needed for the construction of these two curves include the following:

(*a*) Total area within the reach or area in which crop losses are to be computed;

(*b*) The average proportion of the total area that is cultivated;

(*c*) The average acreage devoted to various crops raised in the area;

(*d*) The average cost of preparation of land and drop at various seasons;

(*e*) The average prices of the harvested crops;

(*f*) The dates or seasons of various farm processes performed in the raising of crops (that is, planting, cultivation, harvesting, and plowing); and

(*g*) Latest date on which a crop may be replanted so as to produce a harvest.

Items (*a*), (*b*), and (*c*) are used in the construction of the stage versus area-flooded curve, and items (*d*) to (*g*) are required for the crop-loss curve.

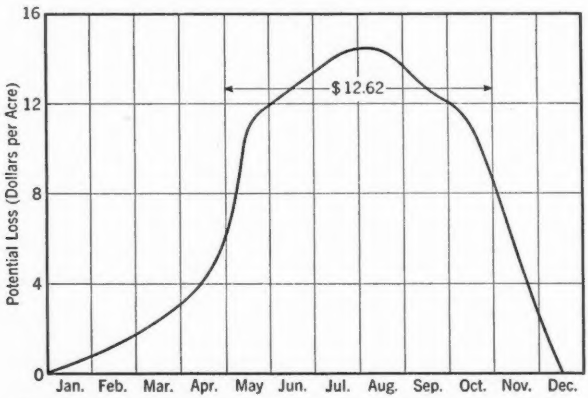

FIG. 4.—POTENTIAL CROP LOSS; REACH ON THE LITTLE RIVER, ARK.

The area-flooded curve is based primarily on the total area of the reach, since the land uses are more easily checked on that basis. Aside from that reason, it could be made to show cultivated area only. A computation of annual loss is given in Table 2.

COMPUTATION OF INDIRECT LOSSES

Indirect annual losses cannot be computed on the foregoing basis, primarily because the relation between the stage and loss is not constant. Other factors enter, which differ with the source of the indirect loss. Business losses, for example, depend also upon the time during which the plant is closed for repairs, which period will vary also with the nature of the business as well as the extent of the physical damage. The cost of rerouting of traffic will depend upon the length of available detours and the time needed to make repairs. Each source of indirect loss contains some other factor to affect the relationship.

There appears, however, a somewhat more constant, closer relationship between total direct and indirect losses of individual business concerns, since greater indirect losses will be sustained when the more extensive damage must be repaired. On the basis of this relation, a ratio of indirect to direct damage may be computed from the field data, adjusted if necessary, and applied to the annual direct losses. For example, the indirect loss in a particular flood may amount to 40% of the direct damage; then the total annual loss will equal 140% of the direct annual loss.

TABLE 2.—Crop Loss for Period from May to October, Reach on Little River, Ark.

Stage, in ft	Frequency, No. of Floods per Yr		Land Flooded, in Acres		Crop Loss		Annual loss
	For stage	For interval	For stage	Mean for interval	Per acre	Total	
26.0	0.608		0				
		0.257		270	$12.62	$ 3,407	$ 876
27.0	0.351		540				
		0.1847		760		9,591	1,771
28.0	0.1663		980				
		0.0992		1,155		14,576	1,446
29.0	0.0671		1,330				
		0.0431		1,465		18,488	797
30.0	0.0240		1,600				
		0.01615		1,710		21,580	348
31.0	0.00785		1,820				
		0.00545		1,895		23,915	130
32.0	0.00240		1,970				
		0.00220		2,055		25,934	57
34.0	0.00020		2,140				
		0.00018		2,160	12.62	27,259	5
36.0	0.00020		2,180				
Total annual crop loss for the period							$5,430

Benefits from Flood Control

The primary source of benefits from protective works is the elimination of flood losses. The benefits are of the same diverse nature as the flood losses, and various means must be used to evaluate all types. Floods are recurrent phenomena, and therefore the risks are continual although the damages are intermittent. The losses, or cost of the risk, are a charge against income; that is, they are an expense which, without protection, must be paid, although it is irregular, and for which nothing is received in return. The benefits are equal to the losses eliminated. In accordance with this principle, the direct annual flood benefits from works giving complete protection will be identical with the total annual losses.

The annual benefits obtaining from works (reservoirs, for example) which may give only partial protection may be computed to the extent that floods are reduced. For example, suppose that a reservoir had been constructed which would reduce all floods at a given point by 1 ft. The benefits result from reducing each flood that may occur and thereby save a part of the possible loss. For a given flood the same causes, or substantially the same in effect, will operate to produce the given peak after the construction of the reservoir, as previously. Therefore, the frequency, expressed as the expected number of floods per year, will remain the same; since the flood is reduced 1 ft, however, the damage will be taken from the stage-damage curve at a point 1 ft lower than the natural stage. The stage at which damage begins will remain the same so that all floods above that point to the limit of the reduction will be reduced below the danger point. The computation of the annual loss is repeated with control (see Table 1(*b*)), and the difference in annual loss under the two conditions is the annual benefit:

Annual loss under natural conditions (Table 1(*a*))	$28,960
Deduct annual loss, with reduction of 1 ft (Table 1(*b*))	12,970
Annual benefits with 1-ft reduction	$15,990

The damage center is the same as that used for natural conditions (Table 1(*a*)).

Reservoirs will not reduce all floods equally. Small floods below the danger point generally will be passed with no reduction. A reservoir usually will be designed to control a specified "design flood," the reduction for which will probably be the maximum since larger floods will pass over the spillway with a smaller relative reduction. A curve will be required to show the reduction obtainable over the full range of floods. The computation of annual loss is substantially the same except that the variable reduction obtained from the curve will replace the uniform reduction. As before, the annual benefit will be the difference in annual losses obtained before and after control.

Levees will eliminate all direct losses caused by inundation up to the stage at which they are overtopped. Beyond that stage the loss is as great as if there were no protection. The annual benefits will equal the annual losses below the elevation of the top of the levee or wall.

Losses and benefits can be computed graphically from a diagram such as is shown in Fig. 5. Values on the ordinate scale are the numbers of floods per year, and on the abscissa, the damage caused by a peak at the corresponding stage. The area under the curve (area *O A B C O*) then represents the total annual loss from uncontrolled floods. Next, a similar curve is constructed for losses after control (area *O D E F O*, Fig. 5). The area between the two curves (area *A B C F E D A*) represents the average annual benefits.

Accurate distribution of benefits is possible by the foregoing analysis. It may be desirable in some cases to determine the benefits accruing from two or more means of protection. It is only necessary to fix the range of stage through which each method of protection and the benefits obtained may be readily computed. The benefits derived by each type of protection will be the reduction of losses through the range of stage in which it operates. It should be remembered that the benefits per uniform increment of stage vary in accordance with the frequency of the floods expected.

In some cases, the benefits from protection of agricultural land are obtained as average value per acre. No objection can be raised against this, provided that it is used only where the total for the given area can serve equally well. It is an erroneous step, however, to assume that the higher areas covered by the greater floods will have the same average annual benefits as the lands at lower elevations. For example, levees may protect to elevation *a*; land above stage *a* will not yield the same average annual benefits with protection as those below for the reason that the latter are more frequently flooded.

Evaluation by Estimating Depreciated Property Values

In the foregoing evaluation of flood losses and benefits, the entire process was predicated on the basis of an annual loss or expense of risk, and therefore a charge against revenue or income. In contrast with this method, evaluation of flood losses has been made on a capital basis, which consisted essentially of an estimate of the decrease of capital value of property caused by one large flood. This process has been used in computing the value of flood protection and hence must be given some consideration.

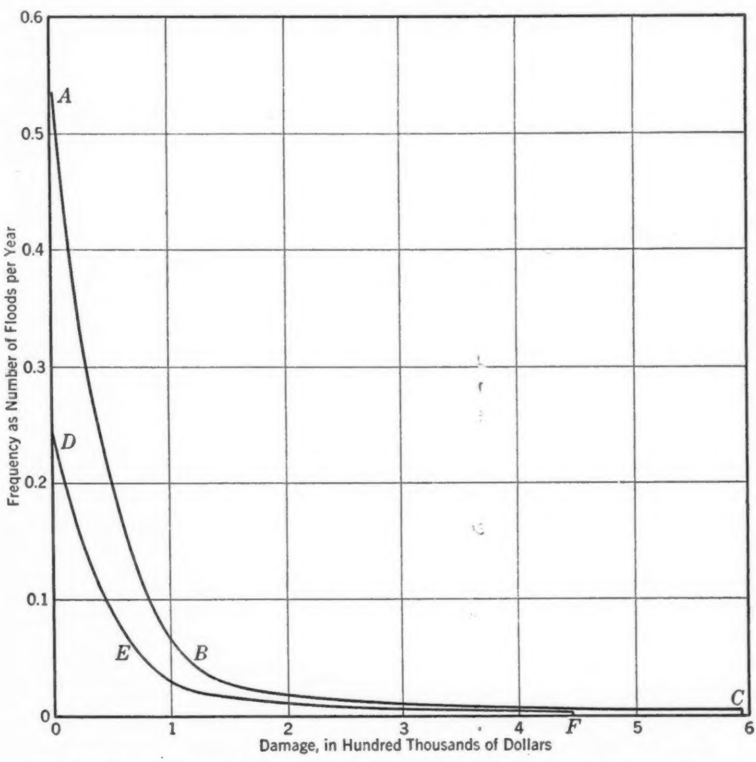

Fig. 5.—Graphic Computation of Flood Losses and Benefits, Lawrence, Mass.

An appraisal is made of the affected property after the flood to estimate its value then and the value as before the inundation. The difference between the two values is taken as the flood loss and, if damage is fully eliminated by protective works, the benefits to be obtained. These appraisals must be considered primarily as a sale value or equivalent since construction costs will not be materially affected by the flood risk of the completed structures. For present purposes it will be assumed that the difference in appraisals includes all the costs.

Except for the effect of the construction costs, which constitutes a factor tending to resist change, the value of property will depend upon the income that can be earned by productive property or upon the desirability for use intended in the case of nonproductive property. This value will be manifest in the sale price of property, which should correspond to the appraised values. If a piece of property becomes subject to a flood loss, its desirability either for profit-making, habitation, or other use becomes less, and its capital value falls. Therefore, the appraised value or sale price will be less. The decrease in capital value, or depreciation of value, is seen to be simply a subjective evaluation of the total annual flood loss in terms of the capital value of the property. (The first expression of this concept heard by the writer was made by Major Hugh J. Casey, M. Am. Soc. C. E., Corps of Engineers, under whose direction he was working at the time. The fitness of the idea makes it of fundamental importance in explaining the relationship of annual loss and depreciation.)

In the case of productive property, the decrease in value must originate from all causes operating to reduce the income or revenue. Necessarily, it includes all indirect as well as direct losses. Personal risk, however, will be small except for the danger that the damage may be so extreme that it may cause bankruptcy.

Depreciation of the value of nonproductive property, other than residential, should be small. Except where floods rise extremely quickly, or in the buildings used to house people unable to escape, the danger to life and, therefore, the personal risk, is also small. Consequently, the decrease in value should be limited to the additional expense of the flood risk or annual loss.

On the other hand, floods in residential areas entail considerable personal risk that cannot be measured in the monetary value of the damage or the cost of relief or evacuation. This personal risk includes general inconvenience, danger to health, and possible loss of life. All this will affect, adversely, the desirability of the area for habitation and thus reduce the value of the property. This personal risk can have only a subjective value, being worth what the individual will pay to escape it. Unfortunately, there is little else upon which to base an estimate of the monetary value of the personal risk except the decrease in property value or the making of some allowance, more or less arbitrary, in the indirect losses. Under the latter assumption, indirect losses of residential areas may perhaps be considered to be the same as those of the business and industrial areas.

There are several serious defects connected with this method of evaluation, however, which preclude its acceptance for general use. Depreciation of value will be quite high immediately after a flood, especially if it is the first experience in the area; but, as the hysteria subsides, property will regain a part of the loss in value, and, as time erases the memory of the flood and new people come into the area, values will probably reach practically their former levels. The decrease in value will depend upon the time elapsed since the last great flood, but the real flood risk will continue as before unless some means of protection have been provided.

The capital-loss method bases the estimated damage on a single flood. As is well known, there are other floods of lesser magnitudes and possibly others of greater, and therefore there is only the faintest chance that the total damage of a single flood can equal the capitalized value of the annual flood loss. The annual flood loss or risk sustained is the total of all floods, great and small—not from just one large inundation. Furthermore, the damage caused by one flood cannot be compared properly with the capital cost of protection because of the element of time, which enters through the frequency of floods and the interest on the investment.

In order to determine the capitalized value of the annual flood loss, the latter must first be obtained by some means, after which there would be little reason to capitalize it. It may be said that the annual cost can be obtained from the total flood loss by applying a suitable rate of interest. What rate? This question must be answered whether the annual loss is desired from the total damage or the latter is obtained by capitalizing the annual loss. In either case the rate must be selected arbitrarily or on a basis not relevant to the flood risk, which action will yield a result little better than a guess.

Other Benefits from Flood Control

Flood-protection works frequently result in benefits other than the elimination of the losses discussed herein. A source of benefit may be found in some localities in the possible appreciation of land values when an area is adequately protected from a recognized flood menace. The lands subject to such appreciation must be situated adjacent to, or in, a highly developed locality where other land is scarce so that there can be little or no doubt about the utilization of the land once it is protected. Appreciation of land values opens a way to assume highly speculative returns as benefits for protective works, and for this reason such possibilities should be examined with great care. Conservative estimates of benefits will eliminate the assumption of increased land values except in the most favorable cases. Before it can be allowed, there should be in view a need for the land and a manifestly ready market, two conditions which practically limit appreciation to favorably located urban property. At least for the present the alleged overproduction of farm products should be a sufficient objection to allowing appreciation of rural lands as a benefit of flood protection.

Occasionally a vast, ill-defined, and unevaluated benefit (designated "social benefit") is advanced as a reason for flood protection. Social benefits are essentially the total elimination of all the foregoing losses, including the personal risk. Except the latter, all such benefits can be evaluated by some means. The value of the personal risk can be evaluated subjectively only, and will depend upon what the community will pay to eliminate it. Certainly a community that has voted against contributing as little as the right of ways has stated in no unequivocal terms that the flood risk, personal or other, has no value for the people living there.

When flood protection is sought by means of reservoirs, other benefits may (in favorable cases) be obtained. In large reservoirs there may be storage for power, or sanitary, or navigational, uses by augmenting the stream flow in low-water season. Pools may be maintained at fixed elevations so that more or less permanent lakes are created, which may be utilized for summer resorts or other forms of recreation. All these, however, are by-products peculiar to one type of protective works, and to certain favorable sites, and will assist in reducing the cost chargeable to flood protection if they have a recognizable value. They are not obtainable by the reduction of flood losses, however, and for that reason are not considered further in this paper.

Collection of Damage Data

The collection of data of flood damage is a task that requires care, experience, and a broad understanding of economic values in all phases of human activity. Much of the data must necessarily be estimated, particularly the indirect losses; and, although there are many cases of actual costs obtainable shortly after a flood, considerable judgment necessarily must still be exercised. Reliable judgment can be derived only from experience. The compiled data should be as full and complete as can be secured with reasonable effort. Information of certain kinds is necessary, and other kinds are desirable for guidance of judgment or basis of estimate.

The essential data required for computing the value of total damage and the annual loss fall into four distinct classes: (*a*) Monetary value of the damage sustained; (*b*) elevation of the property above a fixed datum which is, or can be, correlated with the available data of flood crests; (*c*) location of the damage; and (*d*) stage or discharge of the flood. In addition to the foregoing data, more information is needed to support the primary data of damage. This information is extremely varied, since it pertains to every variety of economic activity, and can be classified as follows—

For buildings: Approximate dimensions, condition, type, and material of construction.

Bridges: Purpose, type, dimensions, material, and approximate age and condition.

Highways and railroads: Type of surface, length affected by the flood, and amount of traffic.

Business losses: Nature of business, approximate payrolls, annual sales or production, and gross profits.

Acknowledgment

The ideas and opinions expressed herein have been developed by the writer while engaged in work on flood-control studies in the various district offices of the U. S. Engineer Department. They have grown with the duties undertaken and have been shaped by innumerable conferences with many colleagues. The opinions expressed herein, however, are those of the author and do not necessarily reflect the official view of the U. S. Engineer Department.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

TRAFFIC ENGINEERING AS APPLIED TO RURAL HIGHWAYS

By Milton Harris,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Highways are built for one purpose only—to carry traffic. In the operation of American rural highway systems two problems have to be faced:

1. To modernize existing roads so that they may meet the demands of efficient transportation.
2. To incorporate in new design those elements that study of existing traffic has taught are efficient factors.

This paper presents various techniques for accomplishing these ends in part.

Introduction

The practice and literature of traffic engineering heretofore have been applied principally to city streets and the environs of urban areas. As a result, investigations have been narrowed to those items that directly affect this type of traffic native to these localities. Corrective measures in general have been confined to erecting signs, parking, placing, and operating traffic signals, channelizing and alleviating conditions that are conducive to high accident frequency at intersections.

The problem of moving rural traffic safely and speedily now demands attention and, in fact, has come to be regarded as of major importance. Its insistence lies in the fact that although the actual number of accidents inside American urban areas is greater, nevertheless by comparison those that occur outside are the more severe and cause a greater number of fatalities.

To fully appreciate the problem that lies before the highway engineer, it is necessary to give some thought to the fundamentals of traffic that produce these troubles. Of two schools of thought, one considers traffic as a homogeneous stream and the other, as made up of individual units to be dealt with singly and individually. Education and enforcement take the latter view,

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by September 15, 1941.

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but engineering has to consider the former. Both activities may function in unison.

Taken as an individual, the motorist indulges in a variety of antics that express his own personality and training. Even he himself cannot realize why he does certain things in a particular manner. Taken as mass behavior, however, such actions are reasonably capable of being measured and valued. Experience shows that this mass action follows certain definite patterns, as affected by each motorist's individual life and surroundings. In certain localities vast numbers drive to work at 7:45 a.m. Each has to be at work at eight, and each leaves his home at a different minute, yet the mass moves down the street at a particular time of day.

This stream of cars proceeds at a certain speed, as regulated by the individual's sense of caution and by his reaction to the surrounding physical features. Most drivers follow a definite pattern that is as fixed as the time at which this mass movement takes place. Because traffic follows such a pattern to a great extent the traffic engineer can define the pattern's extent and may cause it to change by introducing human and physical factors. The human factors are the police with their regulatory functions and the educators with their teaching propaganda. The physical factors are the surrounding terrain, signs, signals, stripes, islands, and kindred design of the highway structure. Whatever the origin, the least change affects the traffic stream.

Traffic psychology is complex and as yet unsolved; although it may be expected that certain obvious changes will affect the stream flow in certain ways. No one knows what effect certain types of buildings, certain visibility, or certain other contiguous traffic streams will have on a particular flow of traffic. This type of research should be undertaken in the near future. It seems strange that so much accuracy in detail of design of highways should obtain, whereas so little is known about the element that will use the road.

By analyzing traffic action, traffic behavior can be determined and evaluated. Roads should be built accordingly; hence, it seems reasonable to expect a road design that will accommodate that behavior which is normal to those who drive with prudence.

If all these deductions are valid, they serve as a basis for further development of the thought that engineering principles may be applied to traffic flow and traffic trouble as well as to the analysis of liquid flow or liquid disturbances. Underlying all, there seems to be a pattern. It is the engineer's problem to identify this pattern and, if an adverse one, to effect a change by altering either the physical structure or the human regulation and education of the traffic stream. Doubtless, driving habits can be changed by education and by enforcement, but more probably can be accomplished by natural regulation resulting from properly designed highways which accommodate themselves to reasonable existing traffic patterns, aided at proper points by efficient regulation rather than by severe enforcement.

In applying traffic engineering to rural highways the most outstanding feature that needs correcting is the present structure, which must be made safe for existing traffic. The crying need is to reduce the large mortality on streets and highways, yet provide the maximum mobility possible for users.

Next in order, it is essential to provide designers with traffic data so that future roads will be the most efficient obtainable with the money available. These two projects should occupy the immediate attention of traffic engineers until the hazards have been brought under control. After that, other needs of traffic should be subjected to engineering examination in the order of their importance. Among the problems requiring consideration are congestion, pedestrian control, freeways, allocation of tax moneys, roadside development, integration of transportation facilities, and many other worth-while projects.

This paper will be limited to two phases of the traffic engineering problem: (1) Modernization of present facilities by investigating hazardous locations; and (2) utilization of traffic data for design.

Points of Hazard

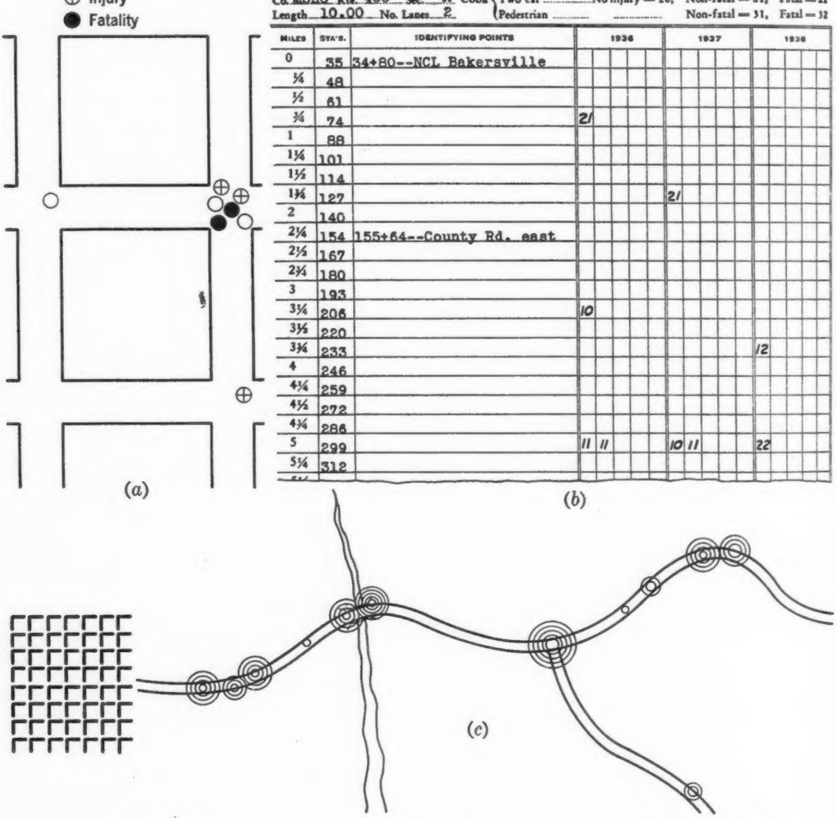
In any engineering investigation the collection of the facts appertaining to the problem is the first consideration. Then comes their arrangement in such manner that they may be studied and pertinent conclusions drawn. In the study of rural accidents, first it must be known (*a*) where, (*b*) when, and (*c*) why the accidents happened; then from these facts logical conclusions can be drawn and reasonable recommendation made for the remedy.

So long as accidents are distributed along a highway and their frequency is proportioned to the density of traffic—or as it is termed, the accidents are proportional to traffic exposure—there is little that can be done in attacking the problem from an engineering standpoint; but let one particular location become the focal point for more than a very few accidents and the engineer has a sample on which to start work. He can analyze the facts for the pattern involved. Not that it is necessary to have accidents at a particular point before an investigation can be started; but the fact remains that ordinarily records are already available, and more are being accumulated, to establish these points of accident frequency as the localities that demand immediate attention. As will be shown later, direct investigation of driver habit can be made in the field without recourse to the accident record, other than its location.

To locate these points of recurrence, so that investigation may be made, "pin maps," "spot cards," or "concentric circles" (Fig. 1) may be used to show where a "pile up" is taking place. Having collected, coded, entered, and filed the accident reports, the most hazardous location on a particular stretch of highway is at once apparent. The next step is to determine "why." This process is usually a comparatively simple procedure in city work, inasmuch as an analysis of the accidents that have occurred generally indicates the solution at once. In rural investigations the causes are more obscure; they are numerous and often are not mentioned in the reports because they were not apparent to the patrol officer at the time. For this reason a different technique is suggested to overcome any lack of information.

This technique is based on the premise that the accident reports reflect certain bad driving habits on the part of some motorists using that particular highway. Due to the changeability of such habits as affected by external influences, it necessarily follows that no two points on a highway will have the same effect. Hence, the investigation must be confined to a relatively small portion or area if the reason is to be determined for a series of accidents at that point.

To indicate those bad driving habits, the accident reports themselves are analyzed for all their elements. Usually one or more outstanding contributory causes motivate the underlying pattern. These are used as indicators with which the traffic engineer arms himself before going out into the field to ascertain the proportion of the traffic that indulges in questionable practices.



(*a*) Pin Map (*b*) Spot Card (*c*) Concentric Circles

FIG. 1.—ACCIDENT INVENTORY

He must use his ingenuity to devise tests to determine these causes. His tests must be extensive and accurate enough to give him results that are trustworthy, because from his results the engineer must formulate a remedy that will exclude habits conducive to accidents, without at the same time introducing other and more severe hazards. It is true that any change may introduce an element giving rise to another habit which in turn may produce accidents, but these accidents should be of a different type. Sometimes it may even be advisable to deliberately introduce into the highway structure a physical change that will convert one type of accident into another less dangerous. For example, a division strip built into a four-lane highway should eliminate head-on accidents, or medial friction, which in rural sections ordinarily results in many fatalities because of the high speeds involved; but a preponderance of rear-end accidents may then occur. However, these are not usually so severe as the head-on type; and by further application of investigative technique, remedies for the rear-end type will be found.

For ease in analyzing accident report data it is customary to list on a summary form the various factors that have been reported for each accident, arranged across the page, one accident to the line, as shown in Fig. 2. (The notation for the driver in the third item as "HBD" is code for "Had been drinking".) By adding up all factors and expressing them as percentages, their relation to each other is noted as one element of the problem, and their relation to state-wide statistics gives immediate indication as to what factors of the problem are worthy of investigation as indicated in Table 1, which relates

TABLE 1.—PERCENTAGE ANALYSIS OF ACCIDENT DATA
Route Mono-400-A; Curve 5.2 Miles North of Bakersville

Item	For This Location		State-wide percentage
	No. of Accidents	%	
Daylight	2	40.0	43.63
Darkness	3	60.0	55.85
	5		
Clear	2	40.0	....
Cloudy	2	40.0	....
Rain	1	20.0	....
	5		
Fatal	1	20.0	....
Non-fatal	3	60.0	....
Property damage	1	20.0	....
	5		
Single car	4	80.0	29.61
Two or more cars	1	20.0	70.39
Pedestrian	0	0.0	....
	5		

Item	For This Location		State-wide percentage
	No. of Accidents	%	
Dry pavement	3	60.0	....
Wet pavement	2	40.0	....
	5		
Speed	5	50.0	16.43
Wrong lane, not passing	1	10.0	11.75
Soft shoulders	1	10.0	0.17
Out of control	1	10.0	....
Had been drinking	1	10.0	13.26
Blinded by lights	1	10.0	2.14
	10[a]		
Two Lanes:			
Passenger cars (single-car accident)	4	80.0	26.77
Passenger car vs. light truck	1	20.0	4.84
	5		

[a] More than one contributing factor may be assigned to accidents.

to the same location as Fig. 2. In fact, comparison of factors, either within the problem or externally with the state-wide data, indicates the accident pattern in many cases. In the example cited, speed is the outstanding contributory cause; hence this can be taken as the indicator for further study in the field.

Checking the field habits against the accident summary analysis is an activity of rural traffic engineering that is in its infancy. It is comparable to devising surveying methods to obtain geographical data for use in design. In

This Summary Covers — Mono-400-A
Curve 5.2 miles North of Bakersville

Motor V

DATE	LOCATION OF ACCIDENT	DAYLIGHT	DARKNESS	WEATHER	TYPES: INJURY			TYPES: ELEMENTS			PHYSICAL CONDITION OF			PHYSICAL FEATURES OF HIGHWAY				TWO-CAR ACCIDENTS—COURSE OF VEHICLES		
					FATAL	NON-FATAL	PROPERTY DAMAGE ONLY	SINGLE CAR	TWO OR MORE CARS	PEDESTRIAN	DRIVER	VEHICLE	PEDESTRIAN	NO. OF LANES	ALIGNMENT	GRADE	CONDITION	APPROACHING	OVERTAKING	INTERSECTION
3/20/36	Station 308+40	X		Clear		X		X						2	Curve	Level	Dry			
9/ 2/36	Station 311+00	X		Cloudy		X		X						2	Curve	Level	Dry			
4/ 4/37	Station 307+90		X	Cloudy			X	X			A HBD			2	Curve	Level	Wet			
10/28/37	Station 306+50		X	Clear		X		X						2	Curve	Level	Dry			
2/22/38	Station 309+85		X	Rain	X				X		B=Blinded by lights			2	Curve	Level	Wet			

Fig. 2.—Tabulati

fact, it is actually making a traffic survey for use in designing a change in traffic behavior. Of many methods, one is shown in Fig. 3, by photography.

In many cases, a field check of those driver habits, suspected of contributing to the hazard of an accident-prone location, will reveal that the "indicators" are erroneous. In this event, a careful study must be made of the traffic flow and every phase analyzed by partial counts until the crux of the situation is found. Many hours of careful scrutiny may be required. It may also be necessary to return many times to observe traffic at various stages of density as well as under varying conditions of light and weather. It is entirely possible to ascertain and isolate those malign traffic habits from observation alone without prior recourse to an accident summary analysis, though, of course, the analysis sheet is a great help as a starter. Sometimes information secured from local residents or from a study of marks caused by accidents is useful in picking out leads that might indicate the patterns sought. Local police or highway officers often are familiar with the data sought and can be of great help if approached in a diplomatic manner.

The next step is to assemble and analyze the field data, and from them draw whatever conclusion is justified. Most of them may be shown in comprehensive form by means of charts or curves, which are valuable in studying the significance of the facts recorded and also useful for inclusion in a report on the problem (Fig. 4). It is advisable to completely analyze all data taken, because in them may lie the answer to a question that is formed in the mind of a person reading the report, which question eventually would have to be answered.

Applying a physical remedy to a traffic danger demands wide knowledge of highway construction procedure and costs. Obviously, it also requires an expert knowledge of the effect that the new construction will have on the traffic

Accident Summary

NOTE.—Code Types of Vehicles as Follows: Pass. Car = 1, Motorcycle = 2, Bus = 3, Light Truck = 4, Truck = 5, Truck and Trailer = 6.

Reported Causes: Improper Turning	Improper Passing	Improper Parking	Improper Signaling	Wrong Lane, Not Passing	Disregarded Blvd. Stop	Skidding	Miscel.	Types of Vehicles Involved (See Note)	Additional Special Items				Description of Accident	Remarks
								1					A northbound at excessive speed lost control on curve, skidded into field on north side of highway and turned over.	
							Soft Shoulder	1					A northbound at excessive speed drove off west side of highway into field.	
								1					A northbound at excessive speed lost control of car and drove off west side of highway	
							Out of Control	1					A southbound drove part way around curve, found he was driving too fast for conditions and skidded off west side of highway.	
								1-4					A northbound at speed excessive for conditions hit B southbound in southbound lane.	

of Accident Data

stream. Even solutions that seem obvious may produce changes in traffic habits that react unfavorably. An example is the use of three-lane pavements over vertical curves with insufficient sight distance.

The physical remedies devised so far might be classified as changes in the geometric design to accommodate prevailing traffic habits, as follows:

(1) Separation of parallel lanes of traffic;
(2) Separation of intersecting lanes of traffic;
(3) Separation of traffic into lanes of varying speed;
(4) Isolation of various turning movements by means of channelizing islands;
(5) Control of movements by means of signs, signals, lighting, and stripes;
(6) Parking provisions.

Other remedies made by the application of human forces may be effected by means of supervision, enforcement, and education. Application of these remedies is within the realm of traffic engineering, but presupposes a knowledge of traffic police methods, modern educational methods, and public relationship. Their use must be predicated on a complete analysis that indicates a stronger probability of success by such measures than by others within the means of the traffic engineer. He is not limited to physical changes. Some have gone so far as to suggest the combining of the administration of all remedial functions into a highway "operating" department.

In applying remedies to traffic conditions, the item of economics is of direct concern to the traffic engineer. Remedial measures may well be promulgated by any layman but their justification often will not "hold water" from the standpoint of money required for their accomplishment. Much research must be conducted before the traffic engineer can evaluate such factors as fatalities

FIG. 3.—TRAFFIC MOVEMENTS TRACED BY PHOTOGRAPHING MOVING HEADLIGHTS. NOTE ONE WRONG ROUTE NEAR CENTRAL LIGHT

and personal injuries, loss of time caused by congestion, cost of car operation when used for pleasure, and others. The answer may lie equally in either engineering or policing, and the final solution may be determined by the relative economies possible. The evaluation of death from the standpoint of insurance risk does not seem to cover the situation except when used on a comparative basis, but the mere totals of deaths or permanent disabilities act as a most potent weapon when waved before the noses of those who must approve the appropriation of funds to correct a hazardous condition.

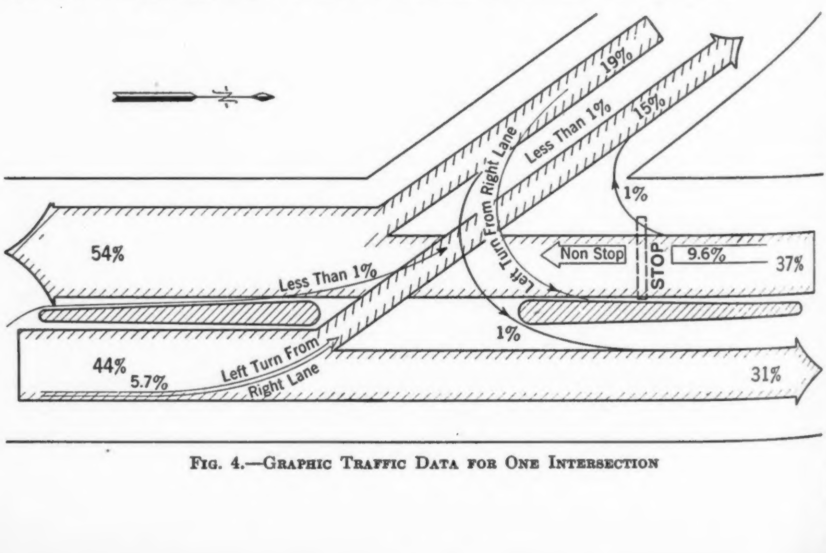


FIG. 4.—GRAPHIC TRAFFIC DATA FOR ONE INTERSECTION

To show the application to practical problems, a few cases will be analyzed in detail.

Case No. 1.—Problem of Dangerous Curve

It will be presumed that a spot card such as Fig. 1(*b*) has been examined, and the location at mile 5 appears to present a hazard that demands attention in relation to other points of accident occurrence on this highway. A perusal of the individual accident reports is made and an accident summary sheet prepared, as shown in Fig. 2. The contributing factors have been summed up and their percentages calculated for purposes of comparison (Table 1) including a partial comparison with state-wide totals.

Those items that exhibit a tendency to depart from the normal expectancy are immediately used as possible indicators—in this case "single cars" and "speed." From knowledge gained through experience and a study of statistics, it is known that in general a preponderance of single-car accidents points either to a road fault or to drunken driving. Speed and road conditions can be checked in the field.

A collision diagram or pictorial representation of the course of vehicles prior to collision (Fig. 5) is a great help in analysis, especially when used with the corresponding summary sheet (Fig. 2). Notations are the same in both. It will be noted that each accident has happened on the outside of the curve, which important fact will find its place in the complete analysis as shown subsequently.

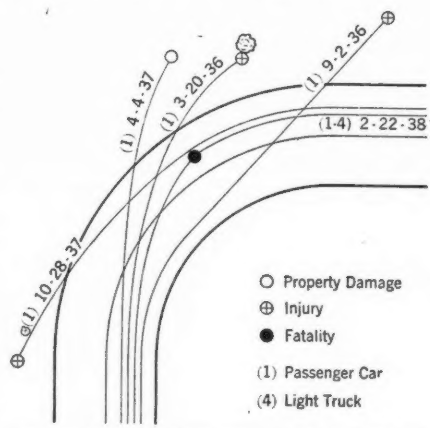


Fig. 5.—Collision Diagram, for Data of Fig. 2 (Illustrative Case 1)

In this problem the field work consisted of making a site plan of the area (sometimes known as a "condition diagram"); then a survey of the highway itself by means of a five-line profile; and finally a series of speed checks to determine the prevailing speeds. The field results are shown in Fig. 6. In the results of one of the speed checks, plotted as a mass diagram (Fig. 6(*c*)), the curve breaks sharply at approximately 85%, indicating that the increments of speeds up to that point have been used by fairly uniform numbers of drivers; but that above this point, there is a rapid falling off. The "break" of the curve has been designated as the "critical speed" and indicates the top speed that the majority of motorists consider safe for that particular spot under the conditions prevailing at the time of test. The entire curve represents the pattern of drivers' habits, whereas the critical speed represents their habit of top safe speed. Obviously, those who exceed the critical speed are the minority, and they are always present no matter how high the critical speed of the sane drivers. A highway capable of accommodating 100% of the driving range could not be financed practicably even if it were otherwise attainable.

Having determined the prevailing speed, the next logical step is to ascertain if the conditions obtaining are safe to accommodate these speeds. It is noted that the speed of approach to the curve is approximately 50 miles per hr. From Fig. 6(*b*) the superelevation is shown to be only 0.1 ft. One formula for centrifugal motion gives the relationship

$$S + f = \frac{0.067\ V^2}{r} \qquad (1)$$

in which: S = rate of superelevation, as a decimal; f = coefficient of friction (0.16); V = velocity in miles per hour (50 miles per hr); and r = radius of curve in feet (500 ft).

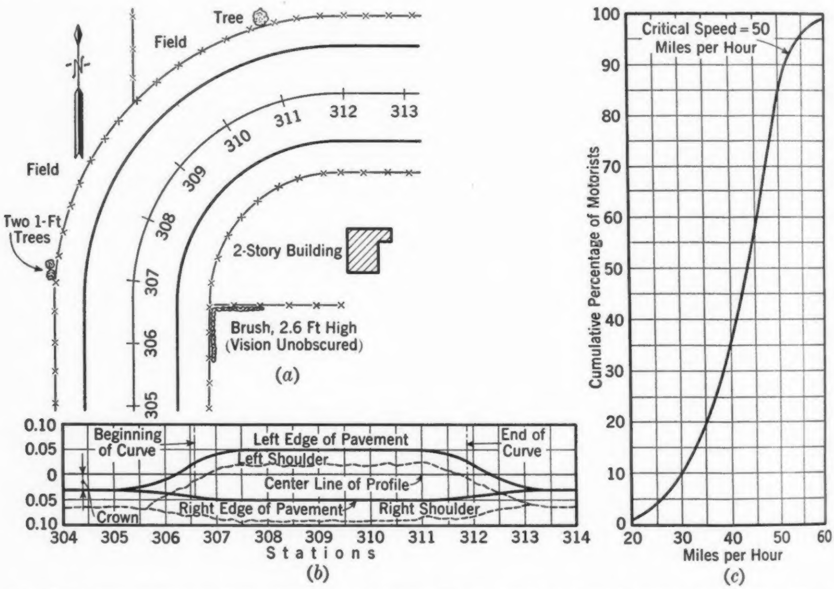


FIG. 6.—SUPPLEMENTARY DATA, INCLUDING (*a*) SITE PLAN, (*b*) FIVE-LINE PROFILE, (*c*) SPEED CHECKS

Applying Eq. 1, it is found that S should be 0.175 as compared with 0.005 obtaining on the ground. This verifies the suspicion that speed too great for conditions prevails at this curve and accounts for the large number of accidents that took a course to the outside of the curve. Even the two-car accident occurred in the outside lane and was evidently caused by the northbound car traveling at too great a speed to keep within its own lane. The single-car accidents occurred on the outside part for the same reason, even though one accident showed drinking as a contributing cause.

Taking the actual S = 0.005 and solving Eq. 1 gives a safe V = 35 miles per hr satisfied by only 19% of the drivers. One may well wonder what happens to the 81% traveling at a speed in excess of that which is theoretically safe. Examination of the traffic habits on the ground showed that a considerable number of drivers applied their brakes upon reaching this curve and in some cases the rear wheels chattered and skidded sideways as the driver attempted to bring the vehicle under control. A test car was able to negotiate the curve at the top speed of 37 miles per hr without undue sway, which checks very closely with the speed by Eq. 1.

The apparent remedy in this case is to change the superelevation. Obviously, a superelevation of 0.175 is too great to accommodate the slow traffic. In addition, weather conditions, types of vehicles, and their loads may be so affected by such a superelevation as to become a hazard in turn. In practice a superelevation of 0.1 ft per ft of width ($S = 0.1$) is about the maximum that will safely provide for all types of traffic. Using this value and solving for R, the problem resolves itself into relocating the curve to a radius of 644 ft with a maximum superelevation of 0.1 ft per ft of width.

As the design speed for the new curve, the critical speed that obtains at the approach was selected; but there is likely to be an increase in this after the new construction is put into operation. If the curve were to be much flattened and the approach environment changed so that traffic probably would increase its speed at this point, then it would be in order to select a higher design speed than is here indicated. Speed checks taken at comparable locations that have been improved to the proposed standards will often furnish reliable data upon which to base judgment as to proper design speed.

TABLE 2.—Summary of Accident Data for Case 2

Description	For This Location		State-wide per-cent-age	Description	For This Location		State-wide per-cent-age
	No. of Accidents	%			No. of Accidents	%	
(a) General Data							
Single car	29	19	32.27	Non-fatal	77	44	....
Two or more cars	121	81	67.73	Property damage	86	49	....
Pedestrian	25	17	6.34	Daylight	70	40	45.27
Fatal	12	7	....	Darkness	105	60	54.56
(b) Contributing Causes[a]							
Driver Condition:				Roadway Condition:			
Had been drinking or drunk	42	17	11.40	Slippery, wet	15	6	3.56
Defective eyes	3	1	....	Impaired sight distance	2	1	4.63
Physical deformity	3	1	....	Obstruction in roadway	1	1	
Asleep	11	4	2.31	Construction under way	3	1	
Fainted	1	1	....	Legal Violations:			
Blinded by lights	3	1	....	Speed too great for conditions	26	11	16.41
Mental case	2	1	....	Following too closely	21	8	3.23
Vehicle Condition:				Improper turning	23	9	5.00
Faulty brakes	5	2	1.61	Improper passing	15	6	6.91
Faulty tires	1	1	2.12	Improper parking	5	2	....
Broken wheel	1	1	....	Wrong lane, not passing	35	14	10.90
Faulty windshield	1	1	....	Disregarded boulevard stop	6	2	....
Broken trailer hook	2	1	....	Skidding	11	4	....
Faulty steering mechanism	1	1	....	Violated right of way	6	2	....

[a] In terms of number of times reported; possibly several different causes for one accident.

CASE NO. 2.—ZONE INVOLVING ACCIDENTS

For a section several miles long, the spot cards indicated a series of accidents greater than adjacent areas on the same route, yet with no particular concentration. The summary analysis sheet (Table 2) indicated that "Had Been Drinking" was the cause that appeared most frequently. Drinking habits are scarcely capable of being analyzed by means of a traffic survey, hence recourse

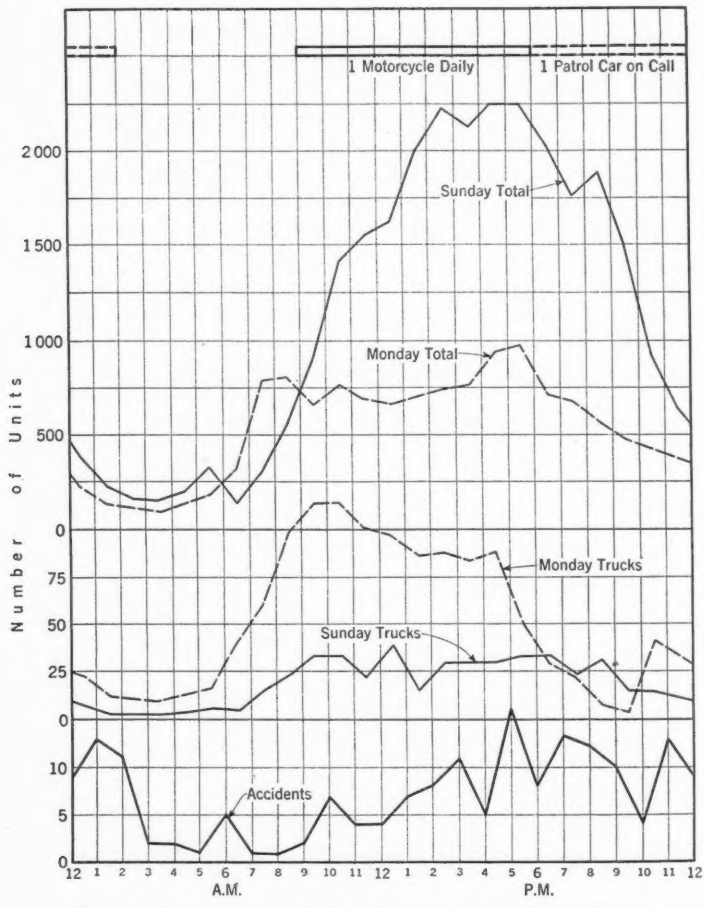


FIG. 7.—GRAPHIC DISTRIBUTION OF TRAFFIC AND ACCIDENTS, CASE 2

was made to another angle of approach. In Fig. 7 the accidents are plotted according to the hour of occurrence. For comparison, the traffic flow is also shown.

From the law of averages, it is reasonable to predict that more accidents will occur during the peak hours of traffic than at other hours, because of the exposure of a greater number of vehicles. The curve shows this to be the case, as between 4 p.m. and 6 p.m. there was a peak in the hourly density curve and a peak in the accident curve. However, it is to be noted that there was another peak in the accident curve at 1 a.m., or when the traffic curve was descending into a trough of minimum density. Obviously a preponderance of drunken-driving accidents occurred at this time in the morning. This was proved further by segregating those accidents that involved drinking.

The remedy in this case lies with the enforcement arm. Further inquiry brought to light the fact that the motor patrol coverage during the peak hour of accidents was only partial (Fig. 7), particularly during the secondary peak at 1 a.m. The coverage on this section of highway therefore should be changed or increased so that adequate patrol would be assured during peak accident hours.

Case No. 3.—Prescribing for a Dangerous Route

Still another case that often arises is partially summarized in Table 3. It will be noted here that 64% of the accidents are rear-end type and 57% of

TABLE 3.—Summary of Accident Data for Case 3

Description	For This Location: No. of accidents	For This Location: %	State-wide percentage	Description	For This Location: No. of accidents	For This Location: %	State-wide percentage
Two or More Cars:				Type:			
Passenger car vs. passenger car	31	41	64.87	Intersection	0	0	29.56
Passenger car vs. truck	43	57	15.18	Approaching	25	33	42.88
Truck vs. truck	2	2	1.23	Overtaking	51	67	27.56
Single Car:				Overtaking Break-down:			
Passenger	14	70	83.80	Miscellaneous (parked car, etc.)	5	9	29.61
Truck	6	30	4.26	Rear end	32	64	44.87
				Sideswipe	14	27	25.52

the total number are between passenger cars and trucks. These are the outstanding indicators of this case.

A short field observation established the fact that this section of highway is a two-lane road with sustained 6% grades, which slow truck traffic down to a comparatively low speed. The passenger traffic is forced to adopt this low speed for long distances because of the lack of safe passing sight distance. The layout of this section is shown in Fig. 1(*c*) with the position of accidents spotted for reference. It is noted at once that the accidents are "piled up" in those sections that are on curves, with a much smaller number on the tangents.

These data suggest that possibly a group of passenger cars are forced to stay behind a slow-moving truck until a driver becomes impatient enough to attempt to pass even without sufficient sight distance. Proceeding on this theory, it appears that possibly the differential in speed between passenger cars and trucks may account for the high percentage of rear-end accidents. This is borne out further by a study of the relative hourly density of trucks and passenger cars (Fig. 8). Therefore, the main line of investigation might well proceed along the line of ascertaining the relation of speeds between trucks and passenger cars and the effect of congestion on this factor.

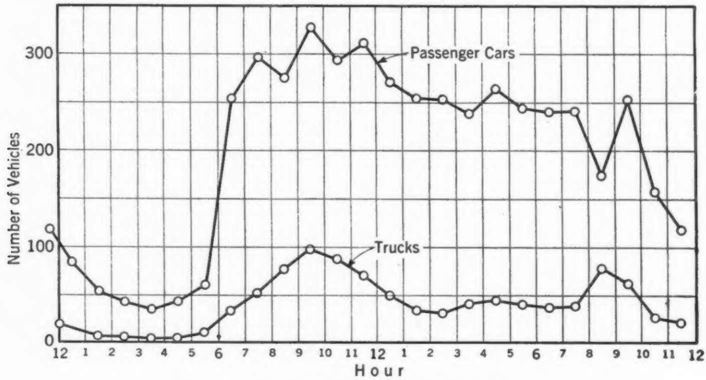


FIG. 8.—PLOT OF TRAFFIC COUNT, CASE 3

To determine the effect of the highway structure and surrounding terrain on the speed of traffic, it is first in order to collect speed data throughout the section, taken close enough together to show the variation in speed, but not taken where congestion exists. The speed, as shown by free-flowing traffic, is the criterion sought. Thus the critical speeds may be plotted on a profile (Fig. 9) and a "virtual speed profile" constructed.

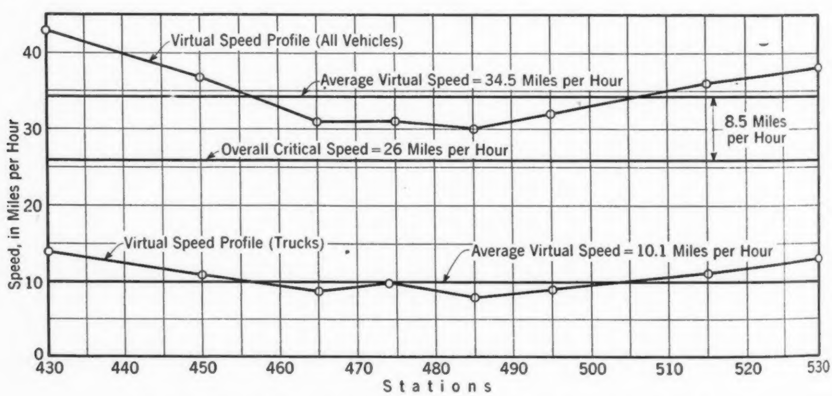


FIG. 9.—SPEED VARIATIONS OVER TWO MILES OF HIGHWAY, CASE 3

The next step is to ascertain the average "over-all" speed, which is the normal speed over this piece of road including time lost by congestion. By plotting a sufficient number of points the usual speed curve is obtained from which a critical speed of 26 miles per hr is selected as the representative safe speed, including time lost by congestion for this section. This has been plotted on the virtual speed profile and indicates that there is a loss of 8.5 miles per hr from the average virtual speed.

A study of the profile indicates those points that will bear further investigation in order that methods may be devised to raise the over-all speed to approach that of the virtual speed. This part of the study consists in the main of separating the slow-speed vehicles from the fast by either additional lanes or a separate truck road. The preponderance of rear-end accidents is added incentive toward separation based on speed; hence, in this case uniformity of speed and safety go "hand in hand."

The average loss of time over this two-mile section was calculated from the data to be 1.08 min or a total over a year's time of 2,222,000 min. This fact has an economic use in selecting the type of design used in the remedy. Various values per minute of lost time have been computed; for example, \$0.0075 for passenger cars and \$0.0147 for an average truck, in one instance,[2] and \$0.01 for passenger cars and \$0.0245 for heavy trucks, in another.[3] Using for this example the value of one cent per lost minute, the total loss capitalized at 5% would possibly warrant the expenditure of \$444,400 for improvement. The proper value to be assigned to the saving of a minute of time is a matter to be decided on in each particular case.

Traffic Data for Design

Design might be briefly described as the accommodation of structure to facts. Therefore, it is logical to presume that all facts are at hand prior to design. Heretofore only scant attention has been paid to traffic facts, as the highway engineer has been engrossed in the immediate problem of providing smooth, dustless roads at a rate consistent with the availability of money. The demand has been ever greater than the supply of funds. Only in late years has he paused for an instant to realize that the very element of his transportation problem—traffic—for which he has been building roads has not only kept up with the rate of production but has so far exceeded it as to create a problem of its own. Fundamental to design is knowledge of the traffic that will use the structure; therefore all traffic facts possible should be gathered for use in design.

Geographical facts are provided by the survey parties, giving the contour of the ground over which the line is laid, the drainage scheme, topographical data, and soil conditions. The laboratory provides facts concerning the materials—their strength, life, and other physical properties. Legal advisers and right-of-way agents determine facts affecting property rights. All these data combined with the engineer's experience form the foundation for his design of a structure to carry the vehicles of motor transportation; but what of the facts surrounding the use of those vehicles on the completed highway?

Securing and analyzing these facts are within the province of the traffic engineer. Inasmuch as this phase of highway engineering has but recently received recognition, traffic data and their relation to design are still in the first stages of evolution. As more thought is given to this subject, so the field will

[2] "Highway Economics," by Sigvald Johannesson, M. Am. Soc. C. E., 1st Ed. McGraw-Hill Book Co., Inc., New York and London, 1931.

[3] "The Economics of Highway Planning," by C. B. McCullough, M. Am. Soc. C. E., and John Beakey, *Technical Bulletin No. 7*, Oregon State Highway Dept.

expand and more and better information will be demanded and obtained; and its analysis will be more definite. Those facts that are readily available at present and have importance in design will now be discussed.

A volume count probably stands first in importance, as the number of vehicles that pass a given point in a certain length of time determines not only the economics of the improvement but the width of lanes and shoulders, the number of lanes and the character of the intersections. A volume count also covers the type of vehicle, the number of passengers—in short, the numerical flow with or without the relative percentages of predetermined subdivisions of type. It is important when making a volume count to have definite knowledge of the use to which that count is to be put in order to gather all necessary data and at the same time to avoid the collection of a mass of information that may be of no value.

The classification of vehicles is of vital importance to the designer if he is to correlate his design to the traffic to be borne. The major classes are usually passenger vehicles and trucks. Each has different elements to be taken into consideration. The ranges of speed are usually different. Their radii of turning are different as well as their wheel loads. Each element has a bearing on design features and the proportion of each classification will affect the design as a whole. Thus, a highway designed to carry trucks alone might conceivably be entirely different from one designed for passenger automobiles. Combine the two classes of vehicles in varying proportions and the design must accommodate the salient features of each classification.

Other types of vehicles would be handled in a similar manner, although their number is usually so small as to be of minor importance in design. The pedestrian, however, although not a vehicle, is a component part of the traffic stream whenever his actions or presence results in conflict. Rural areas are generally free of pedestrian problems except on the outskirts of cities; there it is often necessary to make a pedestrian, as well as a vehicle, count.

For the purpose of design, two elements of a traffic volume count are important. The first is total volume and the second is the peak-hour volume. Total volume for definite periods, such as 24 hr, per month, or yearly, enters into economic calculations. Fluctuations due to season activities sometimes may be handled best on a total monthly basis, especially for comparisons, whereas daily variations have assumed a variety of comparative volumes, based on 24 hr, 16 hr, daily average, daily weekday average, 6 a.m. to 6 p.m. average, and so on, according to the dictates of various engineers' policies. For mutual comparison of these various designated periods it is necessary to know the key hourly percentages to be applied.

The peak-hour traffic is usually the criterion for which the designer plans. By "peak hour" is meant that hour in the daily traffic pattern which most frequently carries the maximum volume during the day. It is the maximum density to be normally expected, yet does not represent unusual conditions prevalent only a few days out of the year—such as traffic for a football game or traffic temporarily detoured over the road in question. Peak-hour density should hold somewhat the same position in traffic engineering when applied to design as maximum stress does in structural engineering.

Successful design must take care of future as well as present conditions. Volumes and densities will change and the designer is obliged to forecast these changes as best he can. Projecting past experience ahead from present knowledge is well established procedure but needs careful manipulation.

In forecasting traffic trends, a straight projection often will lead to fantastic figures. An analysis of past trends will disclose many factors, apparently foreign to traffic, exerting an influence that causes the traffic curve to bend into a form not consistent with past records. Population trends, economic conditions, migration, and even weather conditions have an effect on traffic. In particular locations, public pressure for new roads, improvement of old roads, new industries, and better living facilities, as well as many other causes, are reflected in the traffic curve. For design purposes, the future peak-hour load should be given a great deal of thought and its selection should reflect not only the past trend but also all possible factors that can be estimated as having any effect in the future.

A corollary to volume count, and of importance to the designer, is a record of turning movement. Naturally applicable only at grade intersections, the percentage of total volume that leaves, enters, or crosses the main traffic stream is a definite index of turbulence within the stream. The larger the turning movement, for example, the greater must be the provisions of design to accommodate this factor with as little disturbance of the other traffic units as possible.

Many devices for accomplishing this have been designed and constructed. They range from simple traffic stripe markings to complicated "clover-leaf" intersections. The underlying factors that determine the degree to which an intersection should be improved rest on the total yearly volume for economic reasons; and on the turning and crossing movements, particularly during the peak hour, for traffic reasons. Delays also enter the study of intersection design and are capable of being determined and evaluated.

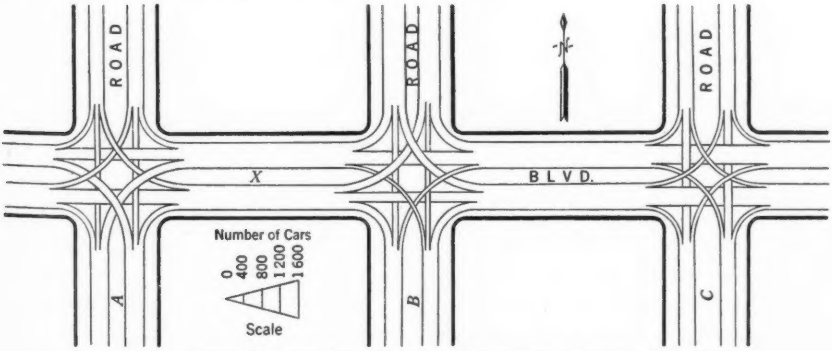


FIG. 10.—GRAPHICAL VOLUME CHART SHOWING PEAK-HOUR TURNING MOVEMENTS

Turning movements may be recorded in the same manner and at the same time that a volume count is made. The hourly percentage of turning movements follows a traffic pattern just as volume passing a point does. Hence, short counts may be expanded by use of key-station counts as a basis. A typical example of the application of volume and turning-movement counts may be seen best by reference to Fig. 10. Boulevard *X* is to be improved and the intersections of *A*, *B*, and *C* roads are also to be improved. It is necessary to ascertain the traffic flow throughout the proposed project and provide the engineer with a flow sheet for his design.

From observation and previous knowledge of the traffic in this vicinity it is decided to set up a key station at the intersection of *B* road and boulevard *X*. The counts consist of volume divided into their respective movements for each hour of the day. The volumes are further classified as to type of vehicles if necessary. The field sheets are then summarized as shown in Fig. 11. During

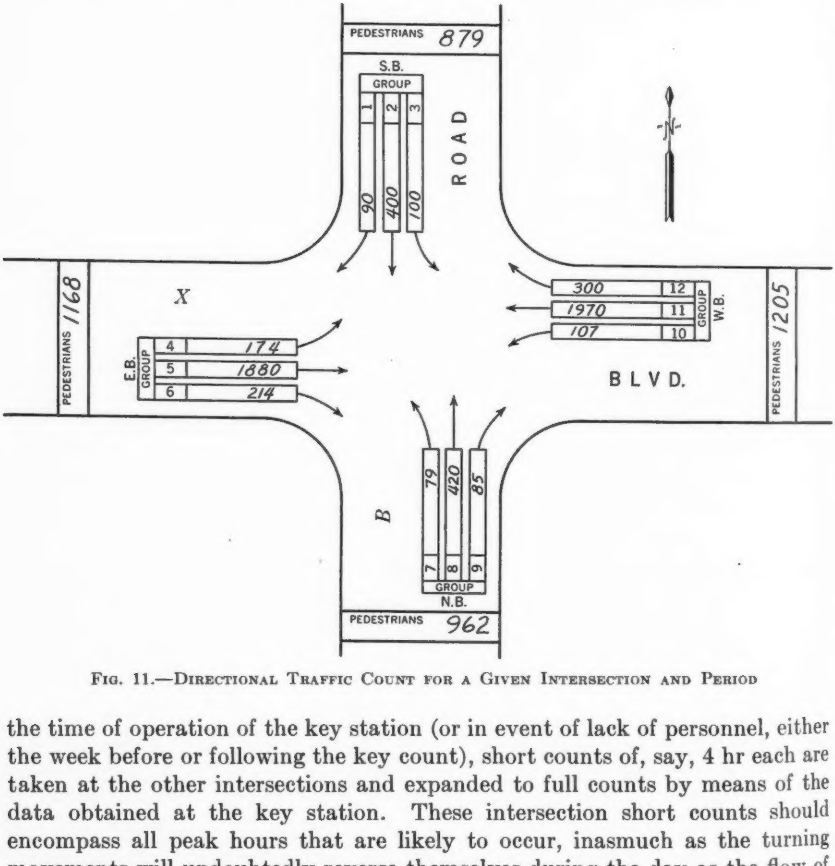


Fig. 11.—Directional Traffic Count for a Given Intersection and Period

the time of operation of the key station (or in event of lack of personnel, either the week before or following the key count), short counts of, say, 4 hr each are taken at the other intersections and expanded to full counts by means of the data obtained at the key station. These intersection short counts should encompass all peak hours that are likely to occur, inasmuch as the turning movements will undoubtedly reverse themselves during the day as the flow of traffic is seldom continuous in one direction.

From these data it is easy to place the facts in graphic form by means of a "flow sheet" (Fig. 10) showing peak hourly flow. Average 24-hr daily flow, Sunday 24-hr flow, daily truck flow, or other relationships may be plotted in like manner.

Speed has lately become the common denominator for design, to which it appears reasonable and logical to correlate most other features. Combined with peak-hour volume and classification of vehicles, these three elements have been adopted by the American Association of State Highway Officials to designate the design limits of any road. Their use is not a matter of guesswork on the part of the engineer. Data to determine their value are collectable and a reasonable figure may be assigned, to which the various elements of a project may be so related that the whole will carry the designated volume at the design speed.

To obtain actual data for design speed, it is necessary to know what speeds are prevalent over the present route. In the case of new construction it is necessary to assume a design speed that will carry the expected traffic expeditiously over the proposed route. Both cases require knowledge of speeds that prudent drivers will use when the highways are completed.

Over longer projects and where special conditions may affect the speed, such additional speed checks should be taken to cover the project thoroughly or at least to permit the plotting of an accurate speed profile. Critical speed is usually accepted as a basis for design speed inasmuch as it represents the maximum speed which prudent drivers, usually 85% to 90%, consider safe. It seems reasonable to design for that percentage of traffic inasmuch as, no matter what design speed is used, a few will drive beyond that limit.

In lieu of information as to what might be expected in speed after the project is completed, speed checks can be taken on comparable roads and the information so obtained may be correlated to design conditions. What the future trend of automobile speeds will disclose is problematical, but past trends may be projected ahead and tempered by judgment. Speed checks taken in flat desert country with the latest type of improved surface, and very light traffic, indicate that the critical speed for automobile traffic will remain under 60 miles per hr. In such localities design speeds of 80 miles per hr and more are obtainable without much additional cost, but it is unlikely that the prudent motorist will exceed or even closely approach such speed as a common behavior.

The selection of a proper design speed contemplates that the engineer will provide a highway over which a goodly proportion of the total traffic will be able to pass safely at, or below such speed. This means that each feature of the road, such as width and number of lanes, shoulders, sight distance, intersections, and surface texture, will allow the same safe and orderly passage. Conversely, any section that does not come up to this criterion is a hazard. To select those factors that affect safe speed, it is only necessary to look at almost any neighboring road built some years ago and analyze each element of design. It is on this basis that speed serves as the common denominator for design.

Intimately connected with speed is the element of sight distance. This is usually divided into two parts—safe passing sight distance and safe non-passing sight distance, the latter being synonymous with safe stopping distance.

The many variables that must be assumed in applying any theoretical formula for obtaining passing sight distance leave the designer with no option but to use whatever values are offered until research has provided more data on which to base values. At present, the tentative values presented by the Design

Committee of the American Association of State Highway Officials are given in Table 4. The approximate formulas have been derived by the writer for "rule-of-thumb" use.

TABLE 4.—MINIMUM SIGHT DISTANCES FOR PASSING AND NON-PASSING

Design Speed, in miles per hour (M.P.H.)	Passing Sight Distance, in Ft		Non-Passing Sight Distance, in Ft	
	By empiricism	By approximate formula, $(M.P.H.)^2/1.6$	By empiricism	By approximate formula, 10 (M.P.H.) − 100
30	600	563	200	200
40	1,100	1,000	275	300
50	1,600	1,563	350	400
60	2,300	2,250	475	500
70	3,200	3,063	600	600

The application of sight distance to design may be divided into two phases, both interrelated: First, horizontal sight distance; and second, vertical sight distance. Horizontal sight distance is affected by such road elements as cut banks, trees, and structures, whereas vertical sight distance is affected by change in gradient which prevents full vision. The ultimate in design is unlimited sight distance, which obviously cannot always be attained. The minimum obtainable bears a direct relationship to safe speed, hence the use of data such as in Table 4. The two types of sight distance are often found at the same location and the minimum is isolated for governing use.

One factor that seems to have escaped the attention of designers is that traffic itself may introduce a blind area. This is particularly true where traffic volume rises to such a peak that an almost continuous stream of cars occupies a lane. In three-lane construction that embraces curvature, sufficient radius must be used to insure minimum sight distance when the outer lanes are fully occupied (Fig. 12).

In the use of sight distance, the designer must cope with an additional factor. Vertical curvature must permit a certain minimum sight distance; and the horizontal curvature, likewise, must be checked so that the driver's line of sight across an inside chord will not be obstructed by a cut bank, a structure, or traffic. To accomplish this calls for ingenuity.

In some cases it may be economy to widen from two or three lanes to a four-lane highway in order to take advantage of a non-passing sight distance inherent in such design as opposed to the longer passing sight distance necessary with fewer lanes. In two-lane construction in mountainous country, the designer is often confronted with a lack of safe passing distance commensurate with his design speed. In such locality advantage should be taken of every possibility to provide at reasonable cost areas of safe passing sight distance spaced at minimum distances apart. The non-passing areas should be "bunched" or localized, yet not be of such a length that an impatient driver will attempt an unsafe passing movement. A fine point in design is presented here, yet from an economic point of view an advantage may accrue in that lesser radii of curvature may be safely introduced in such non-passing areas.

To further protect the motorist, such non-passing areas should be double-striped, signed, or otherwise brought to his attention as areas throughout which it is dangerous to attempt to pass. In general, traffic should be fully warned as to what the road hazards are and should never be allowed to gain a false sense of security. It may seem paradoxical to build a new highway that contains a known hazard, yet no highway can ever be 100% safe nor is it always economically justifiable to build to an excessively high standard for limited traffic. A study of traffic conditions prior to relocation should dictate the amount of the expenditure. It is a sad commentary that highway engineers often allow minority pressure groups to do the dictating.

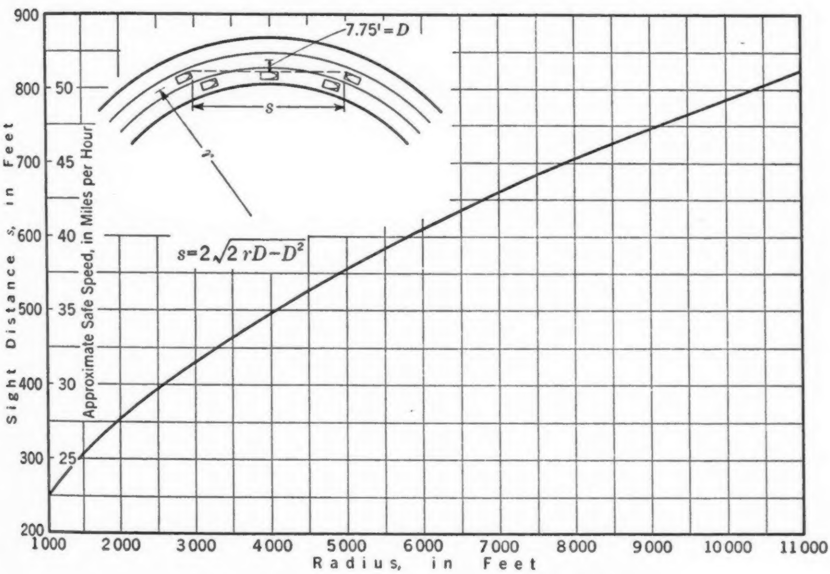


FIG. 12.—SIGHT DISTANCE ON 3-LANE CURVED HIGHWAY

To exemplify the question of sight distance and its relation to design, Fig. 13 presents a study made of a section of proposed relocation designed to a standard speed to be expected for that locality. It will be noted that both horizontal and vertical sight distance have been studied and safe passing sections indicated. Non-passing distance has been isolated and shown in cross hatching. Proper double stripes and signs would safeguard traffic throughout these areas.

Intersection design has received much comment in the technical press because of the innumerable methods proposed to minimize this most hazardous part of a highway. The underlying element is the proper appreciation of the behavior of traffic that will use such construction, as presented in five ways:

(1) Accident history of present intersection;
(2) Traffic flow sheet;

(3) Accident history of proposed design;

(4) Behavior study of present intersection so that proposed design will eliminate hazardous habits; and

(5) Comparative economy in time of passage through intersection using various traffic control devices.

The accident record and collision diagram of a grade intersection should receive careful consideration, as they indicate the intensity of need for change as well as the possible cause of hazard. Accident reports at intersections are much more likely to give a reliable cause than analysis of accidents occurring on the open road. However, the field investigation of driver habit should not be neglected if any doubt should arise as to contributing causes.

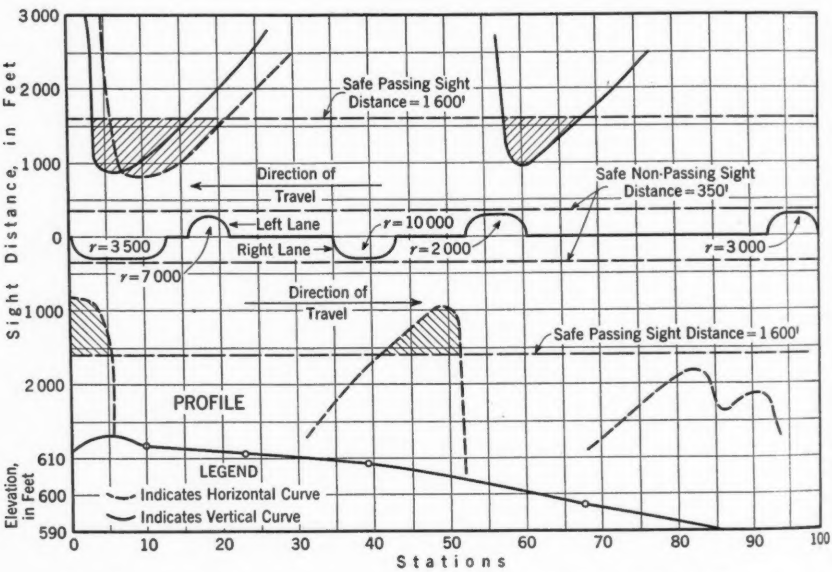


FIG. 13.—STUDY OF SIGHT DISTANCES ON NEW LAYOUT—NON-PASSING AREAS HATCHED

The traffic flow sheet gives the fundamental factor for design requirements and should contain the maximum of information. For instance, if a secondary peak load with heavy turning movement obtains after dark, that fact may have a direct bearing on whether or not the crossing should be illuminated. If the accident history indicates an unusual number of accidents after dark on this movement, the designer would seldom hesitate to specify lighting in addition to special provision to otherwise safeguard the turning movement.

On the other hand, the expenditure of funds to provide a separate right turn when the flow sheet indicates practically none, is over-design. To fail to provide storage space for cars making left turns across heavy traffic is under-design that will lead to criticism as well as to a necessity to provide relief in some other manner.

The accident history of a constructed design, similar to that proposed and under comparable traffic, is invaluable. A very small change in detail may so affect the flow as to eliminate any danger prevailing in the constructed design. It may be that a certain turning movement or a peak volume may be handled in a slightly different manner or that surrounding conditions may be changed to eliminate potential hazards.

The angle of intersection has a direct bearing on design in that the angle of visibility of the driver is confined within relatively narrow limits from within his compartment. It is true that he can probably scan at least 270° of circumference by moving his head and shoulders, but experience indicates that he will barely move his eyes. For this reason, design should accommodate itself to the driver's habits and consideration must be given to the total angle of visibility easily obtainable from an average car. From data provided by General Motors Proving Ground, it was found that the angle of visibility is a maximum when the angle of incidence lies between 65° and 90° (normal to road approached) when approaching from the right.[4] These facts point to design that will cause one stream of traffic to impinge upon another at nearly right angles. The same is true where one traffic stream joins another flowing in the same direction, unless provision is made for acceleration lanes of sufficient length to allow both streams to become equalized as to speed and to commingle. Fig. 14 is an example of design that illustrates this problem in part.

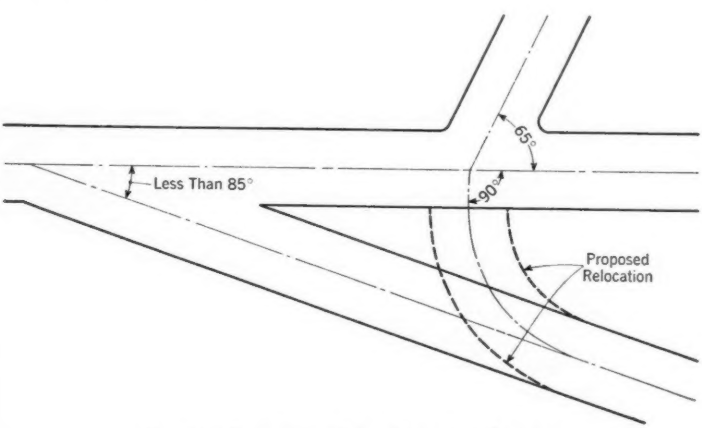


Fig. 14.—Revision to Solve Visibility Problem

The "loss-of-time" element in intersection design is of vital importance, for the efficiency of movement may be measured in terms of volume and time. The efficient utilization of an intersection is generally much lower than the intervening roadway, hence a proportionately larger amount of time and energy should be devoted to intersection design in order to decrease the overall time loss and consequent discomfort to the motoring public. That the average motorist is willing to pay for his comfort is axiomatic.

[4] *Civil Engineering*, August, 1939, p. 494.

Accident records and their analysis should become a part of the fundamental knowledge of a modern highway designer. This is as necessary as knowing the rudiments of a mass diagram. The result of his work should be judged in the light of its ability to carry traffic safely and efficiently. Safety can only be judged by a man experienced in visualizing the effect of all attendant conditions upon the expected traffic stream and asking himself, "What type of accident will this condition impose?" Efficiency of flow must be judged in the same manner, yet both elements depend upon the designer's knowledge of the effect of his labors upon the major fundamental highway factor—traffic.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

PILE-DRIVING FORMULAS

PROGRESS REPORT OF THE COMMITTEE ON THE BEARING VALUE OF PILE FOUNDATIONS

FOREWORD

For several years, the Committee on Bearing Value of Pile Foundations has been engaged in the preparation of a Manual of Engineering Practice. In the course of this work, many phases of pile foundations and bearing value of pile foundations have been discussed in a manner that is comprehensive and in many phases new. Final conclusions have not yet been reached on the subject of pile formulas and pile tests. Since some of the widely used formulas for determining bearing value of a pile were devised more than forty years ago, there have been many developments in the use of piles. Formerly the wood pile was about the only type in extensive use, and some of the original formulas were devised with that pile in mind. Since then concrete piles of large size, weighing many times as much as wood piles, have been used extensively. Even more recently steel pipe piles and steel piles of H-section have been widely used. Pile-driving equipment has undergone almost revolutionary changes since the day of the gravity hammer which was in use almost exclusively when the first formulas were developed. Steam hammers, double acting, and single acting, have replaced the gravity hammer in much work.

The new science of soil mechanics has appeared on the scene within the last two decades, and has added much to the knowledge of foundations in general, and to a new conception of a foundation problem. The engineering profession as a whole has not kept pace with the most recent developments, and it is found that formulas devised forty and fifty years ago are still in extensive use, being incorporated in handbooks, specifications, and engineering literature. The help and usefulness derived from these formulas cannot be denied in view of the many years in which they have been in use. The result is that so many engineers are dependent on the older methods that the introduction of new methods involves two steps: (1) Placing information before the engineers for a discussion by the profession; and (2) as a result promoting self-education on the part of the profession as a whole preparing for the adoption of new methods, should they be found desirable as a result of discussion.

It is from this standpoint that the Committee has prepared outlines of two points of view from which this subject may be regarded. These are pre-

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **September 15, 1941.**

sented herein as Report *A* and Report *B*. The subject is one that is worthy of intensive study, based as much on experience with pile driving as on methods of design, and the theory which underlies application of pile formulas.

It must be realized at the very beginning that the soil is just as much a part of the structural design as is the framework or floor system of the building, or bridge. Although there is much definite information in regard to building materials in common use, and although designers can rely on their structural strength and performance within narrow limits, they do not have similar knowledge in regard to the soil. Even if samples are taken and tested by the most modern methods, such samples refer only to the particular spot where they were found. The soil must be proven or assumed to be the same as the sample; otherwise additional samples must be taken. Here is a risk that the designer and builder of a pile foundation must take. The pile-driving results must be interpreted as the work proceeds and modification must be made accordingly. It must be emphasized urgently that the total of the bearing value of individual piles, no matter whether determined by formula or by load test, is not the bearing value of a pile foundation as a whole. The problem in regard to the pile formula is to determine the bearing value of an individual pile. It must be stated further that the bearing value of a pile determined from the penetration per blow of the last few blows, or even the bearing value determined from the load test, is the value only at the time that the data were obtained. The bearing value of the pile under certain conditions may be something quite different 24 hr later.

It is hoped that all persons who can possibly do so will contribute to this discussion, in the hope that a clearer understanding will result. It is desirable that actual data determined from pile driving, application of formulas, and load tests be submitted, together with a description of the soil conditions existing, all to the end that this subject may be resolved sufficiently to provide a chapter on pile-driving formulas in the proposed Manual of Engineering Practice.

REPORT *A*.—PILE FORMULAS

Introduction

A-1. Definition.—A pile formula is an equation for determining the bearing value of a pile; that is, the resistance of a pile to movement in relation to the soil. Such formulas may be divided into two classes: First, dynamic formulas, determining the resistance to rapid penetration based on the energy of a falling hammer; and second, static formulas, wherein the static resistance is calculated on the basis of certain earth pressure theories or is based on the frictional resistance as determined by tests. These formulas are numerous and of varying validity.

A-2. Limits of Formulas.—In general, formulas for determining the bearing value of a pile are approximations that apply only within certain limits and conditions. Unless these limits and conditions are thoroughly understood by the engineer, and unless the conditions existing at the site are known and will continue after the driving of the pile, the use of a pile formula may be dangerous. It may happen to yield correct results; it may give values that are inadequate for the conditions at the site; or, it may result in the excessive use of material.

A-3. Effect of Soil Types.—Any dynamic formula, however perfect, based on the resistance to driving can give simply the resistance to rapid penetration so that only under conditions where the resistance to rapid penetration is some measure of the static resistance can any such formula be of real value. In sands, gravels, and relatively permeable, incompressible soils, the resistance to rapid penetration is about the same as the resistance to slow penetration. In such soils a sound dynamic formula can give satisfactory results. In clays, silts, and similar cohesive, impermeable, compressible soils, the resistance to rapid penetration may bear little or no relation to the resistance to slow penetration. Usually the driving resistance in these soils is less than the static resistance (in rare cases the reverse is true) and the resistance to re-driving, after the soil has gripped the piles, may be greater than the static resistance.

Dynamic Formulas

A-4. General Formula.—The mechanics of pile driving is rather complicated; it is a phenomenon of the transformation of energy. A basic equation for the resistance of a pile during driving may be derived, using the following nomenclature:

R_d = dynamic resistance;
R = allowable load;
W = weight of striking parts of hammer;
P = weight of pile as driven;
h = height of fall of hammer;
s = penetration of pile per blow;
E = modulus of elasticity of pile as driven;
A = cross-sectional area (average) of pile as driven;
L = length of pile as driven;

n = coefficient of restitution;
e = efficiency of hammer = 0.75 for usual drop hammer = 0.90 for single-acting steam hammer;
$2\,C$ = elastic compression of driving cap;
$2\,C_1$ = elastic compression of pile as driven;
$2\,C_2$ = rebound of pile due to elastic compression of soil and other causes;
$2\,k$ = total rebound of pile hammer; and
$e\,W\,h$ = available energy at instant of impact.

The total energy loss due to elastic compression of the soil, pile, etc., is

$$R_d\,(C + C_1 + C_2) = R_d\,k \qquad (1)$$

The energy loss in impact between hammer and pile, as derived in works on mechanics for the impact of two bodies, is

$$e\,W\,h\,\frac{P\,(1 - n^2)}{W + P} \qquad (2)$$

Therefore, the general equation is

$$R_d\,s = e\,W\,h - e\,W\,h\,\frac{P\,(1 - n^2)}{W + P} - R_d\,k \qquad (3)$$

Every pile-driving formula may be derived from this general formula (Eq. 3) by making certain assumptions regarding the values of the various factors.

A-5. Hiley's Formula.—Solving Eq. 3 for R_d gives

$$R_d = \frac{e\,W\,h}{s + k} \times \frac{W + n^2\,P}{W + P} \qquad (4)$$

which involves no simplifying assumptions. This formula was proposed by A. Hiley.[1]

A-6. Engineering News Formula.—Assuming that the impact is perfectly elastic (an unwarranted assumption), $e\,W\,h\,\frac{P\,(1 - n^2)}{W + P} = 0$; and, solving Eq. 3 for R_d, gives

$$R_d = \frac{e\,W\,h}{s + k} \qquad (5)$$

Now, applying a factor of safety of 6, measuring h in feet and s and k in inches, assuming $k = 1.0$ (actually k may vary between 0.1 and 5.0), and letting $e = 1.0$

$$R = \frac{2\,W\,h}{s + 1} \qquad (6)$$

The foregoing assumptions, made in substituting constants for widely varying factors, result in considerable variation between driving results and actual resistance to penetration.

[1] "Pile-Driving Calculations with Notes on Driving Forces and Ground Resistance," by A. Hiley (Theory and practice; table of forces transmitted through pile; energy requirements; bearing qualities of ground), *Structural Engineer*, Vol. 8, 1930, pp. 246–259, 278–288.

A-7. Coefficient of Restitution.—The driving of the usual types of piles with customary equipment permits making certain logical simplifying assumptions which do not greatly change the result. The energy loss due to impact is a function of the factor n, the coefficient of restitution. Values for n based on Hiley's experiments (which agree closely with those given by E. Nöe and L. Troch[2]) are as follows:

Steel hammer striking steel anvil; steel and concrete pile	0.5
Cast-iron hammer striking concrete; pile without cap	0.4
Cast-iron hammer striking steel plate on wooden cap	0.32
Cast-iron hammer striking good wooden cap or sound head of timber pile	0.25
Hammer striking deteriorated wooden cap or pile head	0.0

A-8. Simplification of Hiley's Formula.—Usually the blow of a pile hammer strikes on the more or less deteriorated wood of either the pile or a driving cap. Under such conditions n would be less than 0.25. Since n is squared in Eq. 4, the value of $n^2 P$ may usually be neglected, therefore, and the assumption made that $n = 0$, as for perfectly inelastic impact. Making this assumption in Eq. 4,

$$R_d = \frac{e\,W\,h}{s + k} \times \frac{W}{W + P} \qquad (7)$$

Applying the usual values of e (0.75 for a drop hammer and 0.90 for a single-acting steam hammer), measuring h in feet and s and k in inches, and using a factor of safety of 3, the following values are obtained: For a drop hammer,

$$R = \frac{3\,W\,h}{s + k} \times \frac{W}{W + P} \qquad (8)$$

and, for a single-acting steam hammer,

$$R = \frac{3.6\,W\,h}{s + k} \times \frac{W}{W + P} \qquad (9)$$

In Eqs. 8 and 9 all the terms are known except k, which equals $C + C_1 + C_2$. The magnitude of k may be evaluated as follows:

(*a*) Assuming the extreme case of end bearing,

$$C_1 = \frac{1}{2}\frac{R_d\,L}{E\,A} = \frac{3}{2}\frac{R\,L}{E\,A} \qquad (10)$$

(*b*) The value of C varies with the type of driving cap, its condition, and the driving resistance. For medium driving resistance (1,000 lb per sq in. on the point of the pile) Hiley gives values of 0.05 in. for timber piles and short driving blocks, and 0.025 in. for $1\frac{1}{2}$-in. wooden pads on concrete piles, with correspondingly larger values for greater driving resistance. It seems reasonable to assume that $C = 0.05$ in. for the heads of timber piles and for short incased blocks, for the usual driving resistance.

[2] "Pieux et Sonnettes," by E. Nöe and L. Troch (Wood, metal, and concrete piles and pile driving), published by Gauthier-Villars, Paris, France, 1920.

(*c*) The value of C_2 is ordinarily indeterminate and it is neglected, but this is probably largely compensated for by the assumption of end bearing in the determination of C_1.

(*d*) Therefore, from the assumptions in Steps (*a*) to (*c*),

$$k = C + C_1 + C_2 = 0.05 + \frac{3}{2}\frac{R\,L}{E\,A} + 0 \quad \ldots\ldots (11)$$

Values of k in inches for different types of pile of varying lengths and bearing capacities are the following:

A-9. Value of h and R.—Solving Eq. 4,

$$h = \frac{R_d\,s - R_d\,k}{e\,W} \times \frac{W + P}{W + n^2\,P} \quad \ldots\ldots (12)$$

It is evident that, for any given case, there is a minimum height and fall, h_0, which is necessary to cause the pile to move; that is, $s = 0$. Using this value of s in Eq. 12, yields

$$h_0 = \frac{R_d\,k}{e\,W} \times \frac{W + P}{W + n^2\,P} \quad \ldots\ldots (13)$$

which, when substituted in Eq. 12, gives

$$h = \frac{R_d\,s}{e\,W} \times \frac{W + P}{W + n^2\,P} + h_0 \quad \ldots\ldots (14)$$

Therefore,

$$R_d = \frac{e\,W\,(h - h_0)}{s} \times \frac{W + n^2\,P}{W + P} \quad \ldots\ldots (15)$$

Now, assuming $n = 0$ (for the reasons previously stated), substituting the value of e, measuring h in feet and s in inches, and applying a factor of safety of 3, the following are found: For a drop hammer,

$$R = \frac{3\,W\,(h - h_0)}{s} \times \frac{W}{W + P} \quad \ldots\ldots (16)$$

and, for a single-acting steam hammer,

$$R = \frac{3.6\,W\,(h - h_0)}{s} \times \frac{W}{W + P} \quad \ldots\ldots (17)$$

The values of all the terms are known except h_0, which is a linear function for any given resistance. It may be determined readily in the field when using a drop hammer. Measuring the penetration for several blows at three or more different heights of drop and plotting the penetration as abscissas and the drop as ordinates, and drawing a line through them, the intersection with the y-axis gives the value of h_0. If the points do not lie in a straight line there has been a change of resistance and the test should be repeated. Note that the value of h_0 varies with the weight of the striking parts of the hammer and with the type, weight, area and length of the pile, so that h_0 should be determined whenever there is a marked change in these factors. Note also that h_0 varies with the square of the dynamic resistance (Eq. 13), and therefore it should be determined when the pile is near its ultimate penetration or probable desired resistance. An illustration of this is offered in Paragraph *A-10*.

A-10. Check of Rebound.—The determination of h_0 permits the verification of the theoretical values of k. Eqs. 3 and 16 give

$$k = \frac{h_0 \, s}{h - h_0} \quad \text{.......................} (18)$$

To check Eq. 18, measurements were made during the driving of greenheart piles in sand and gravel at Boulogne sürmer, France, with a drop hammer weighing 3,000 lb. The value of h_0 was determined, as described, for four different piles. In each case it was 30 cm, or one foot. This gave a value of k by Eq. 18 equal to 0.07 in. The theoretical determination by Eq. 11 gave a value of 0.12 in., thus indicating that Eq. 18 gave conservative results.

A-11. Miscellaneous Dynamic Formulas.—Additional dynamic formulas for the bearing capacity of piles are sometimes used. When the symbols are changed to agree with those in Paragraph *A-4*, these formulas are the following: J. F. Redtenbacher's formula, which is used extensively in Europe, is

$$R_d = \frac{A \, E}{L} \left[- s + \sqrt{s^2 + \frac{2 \, W^2 \, h \, L}{E \, A \, (W + P)}} \right] \quad \text{..........} (19)$$

This assumes perfectly inelastic impact and s is the penetration under the last blow of the hammer.

Franz Kreuter's formula[3] is

$$h = \frac{s \, (R_d)}{W} + h_0 \quad \text{......................} (20)$$

The meaning of h_0 in Eq. 20 is the same as that in Paragraph *A-9*.

A-12. Conclusions.—A study of the mechanics of pile driving and of the assumptions made in the derivation of the several driving formulas indicates that Eqs. 8, 9, 16, and 17 are based on reasonable assumptions and should give reliable values for the dynamic resistance, for all cases within the validity of the assumptions. Comparisons of theoretical results with loading test data tend to confirm this conclusion. For very heavy piles, special driving caps, etc., the values of e, n, and C should be substituted in Eqs. 4 and 15. It is also well known that "whipping" of wooden or light steel piles when being driven with the ordinary heavy hammers now used will have a great effect upon the computed dynamic resistance.

Static Formulas

A-13. General.—In clays, silts, and similar soils—a dynamic formula being frequently unsatisfactory—attempts have been made to derive static formulas. The most frequently mentioned one is that of M. Benabenq,[4] for which very complete tables have been calculated. H. Dörr derived a similar formula.[5] These formulas are all based on Rankine's theory of conjugate stresses. There-

[3] "A New Method for Determining the Supporting Power of Piles," by Franz Kreuter (Mathematical formulas for wooden piles), *Minutes of Proceedings*, Inst. Civ. Engrs., Vol. 124, 1895–1896, Pt. 2, p. 373. Abstract in *Railway Review*, Vol. 36, May 9, 1896, p. 262.

[4] "Calcul des dimensions et du Pouvoir Porteur des pieux de fondation," by Ch. Dantin, *Le Génie Civil*, January 27, 1912, p. 246.

[5] "Die Tragfähigkeit der Pfähle," by H. Dörr, *Bautechnik*, Vol. 10, 1932, pp. 447–450. "Influence de la Forme sur La Resistance des pieux flottants dans les terrains incompressibles ou décompressibles," by Henry Lossier (Supporting power of piles; discussion of tests; formulas; merits of Franki piles), *Le Génie Civil*, Vol. 99, July–December, 1931, pp. 258–260.

fore, they assume that the pressure on a pile will increase directly with the depth. Actual tests show that this is not true, and thus these formulas are not valid. Present knowledge of soil mechanics does not yet permit the derivation of a general static formula, but it may be hoped that research may soon provide one. In any event, even in the use of dynamic formulas, one must bear in mind the necessity of considering static conditions also.

A-14. Approximate Formula.—An approach to a static formula is one based upon friction along the lateral surface of the pile with the unit friction value derived from tests. Assuming a factor of safety of 3, letting A_s = area of the friction surface, in square units, and f = the value of friction per square unit, Eq. 19 takes the form:

$$R = \frac{A_s f}{3} \qquad (21)$$

The magnitude of f varies widely and must be determined carefully in each case, bearing in mind that it may increase or decrease with the depth. Methods of obtaining these values have already been described. The records of the results of certain published tests are given in Table 1.

TABLE 1.—REPORTED VALUES OF f

Location	Soil	Penetration, in ft	Friction, f, in lb per sq ft
Aquia Creek, Va.	Fluid mud	40 to 50	95
New York, N. Y.[a]	Mud	50	130
Rhine Valley	Soft muck	0 to 35	130
Rhine Valley	Soft clay	25 to 35	295
Proctorsville, La.	Mud, sand, clay	35	370
Portland, Me.	Soft blue clay	14	740
Hull, England	Stiff blue clay	18	1,850
Shanghai, China	Silty, microscopic sand	50	130 to 900
Tunis, Africa	Soft, muddy clay	100	370
Zwijndrecht, Holland	Fill, soft clay, peat, sand	40 to 60	600 to 900

[a] Seventeenth Street, North River.

TEST PILES, TESTS, AND SERVICE RECORDS

A-16.—The purpose of the following paragraphs is to present a body of knowledge containing the experience of the engineering profession gained by pile tests and by the observations of the behavior of structures in different localities. Such records should be of permanent value and should serve as a guide in solving pile foundation problems for future construction in other localities where somewhat similar conditions obtain. The cases given are intended to bring out typical conditions of both satisfactory and unsatisfactory foundation construction, the foundation problems involved, and the behavior of the completed structures.

A-17.—A test pile is one on which a known load is placed in order to determine its load-bearing capacity and settlement. The purpose of a test may be (1) to guide the engineer in the selection of a proper type of pile and its load-bearing capacity, or (2) to confirm his assumptions in a design already prepared and to check the quality of the work. In the first case the test piles are specially driven and tested well in advance of the actual construction in order to serve as a basis for the design. In the second case the engineer, having already selected the type of pile and the assumed design load, selects certain piles for loading test purposes from those driven on the actual construction work. The test piles in this case are in essence a check on the design and on the builder's work. When unexpectedly poor results are found they constitute the reason for re-design or rejection of work installed.

A-18.—During driving and testing, careful records of each pile should be kept.

A-19.—The plan may designate certain piles as test piles, or it may be specified that the engineer will select a certain number or percentage of the number of piles on the site as test piles. The selection would naturally be distributed so as to give the most general information. The driving of friction piles selected for test should be recorded fully. The usual method is to record the number of blows per foot at the start and then, as the resistance increases with depth, the number of blows per smaller unit until it has been demonstrated that the pile has reached the specified resistance of the given number of blows per inch. Test piles may then be subjected to load, singly or in groups.

A-20. Methods of Loading.—A suitable platform supported on the test pile or piles may be loaded with pig iron or other measured weights, or, where existing structures of sufficient size are conveniently located, jacking against such dead weight may prove to be more economical. If the jacking reaction is against a beam anchored to two neighboring piles, these piles should each be at least 10 ft from the pile under test. Hydraulic jacks, properly calibrated, give accurate measurement of the loads applied. Friction piles should be loaded to the assumed design load and the settlement should be noted. This load should be fully maintained for a period of at least 24 hr after the pile has come to rest. The load should then be increased by 50% and fully maintained for another 24 hr after the pile has again come to a position of rest. No pile that settles progressively under the full test load can be considered satisfactory.

A-21.—Record of the rebound should be made as the test load is removed. The settlement that may be acceptable under load is somewhat dependent upon the character of the structure to be carried. A total net settlement (after deduction of the rebound) of not more than 0.01 in. for each ton of applied test load, has been deemed satisfactory in some cases. End-bearing piles should be driven to minimum practicable penetration per blow or even zero penetration, provided the pile is not being damaged, and the measured settlement under load should then compare closely with the elastic deformation of the pile figured as a column, with practically full recovery after removal of the load.

A-22.—The recommended method for determining the settlements of test piles is by means of a good engineer's level and a target rod with vernier reading to thousandths of a foot. Two solid bench marks should be established near the test pile. If Ames dials or other strain gages are used, or if wires are stretched alongside the test pile, the fixed supports for these wires or gages should be far enough removed from the test pile so that the readings will not be affected by any settlement of the ground surface that may occur in the immediate vicinity of the test pile.

A-23.—The load settlement curve for a single test pile shows the results of point bearing and side friction as resisted by the soil for that one pile alone. It is no indication of the load-settlement curve for a group of piles which includes the individual test pile; neither is the load-test curve for a group of piles an indication of the load-settlement curve for an entire foundation composed of groups of piles. The elastic compression of the material in the pile itself may have an effect on the load-settlement curve, particularly with long piles. In case the pile, the group, or the foundation load is carried on an unconsolidated layer of cohesive material, then the load-settlement curve for a short time is no indication of what the long-time load-settlement curve may be. The rate of consolidation of such a layer depends upon several factors, including the thickness of the layer and the route over which water may be forced out.

REPORT *B*.—PILE FORMULAS AND PILE TESTS

B-1.—Pile formulas and pile tests have the purpose of determining the bearing capacity of a pile in relation to the shearing strength of the soil immediately surrounding and underlying the pile. It should always be kept in mind that pile formulas and pile tests cannot yield information on the magnitude, distribution, and time rate of the settlements for an entire structure. The actual load on the piles may be only a small fraction of their ultimate bearing capacity and yet the structure as a whole may undergo excessive settlements due to volume decrease of compressible soils underlying the points of the piles. Such settlement may be uneven, corresponding to varying thickness of the compressible layer.

B-2.—Formulas for determining the bearing capacity of piles may be divided into two general classes: (*a*) Dynamic formulas, and (*b*) static formulas. There are many formulas of each type in existence but the dynamic formulas are more commonly used.

B-3. Dynamic Formulas.—The purpose of a dynamic pile-driving formula is to determine the bearing capacity of a pile by calculations based on the resistance the pile offers to driving. Practically all of the available dynamic formulas are based on energy methods, and the fundamental assumptions under which the formulas are derived are concerned largely with the energy losses that occur during impact. The great number of pile-driving formulas that can be found in engineering literature is an indication of the wide variety of assumptions that have been made concerning these energy losses.

B-4. Assumptions.—Some formulas have been based on the assumption that pile driving is a problem in Newtonian impact and that the energy losses can be accounted for in terms of the inertia of the pile and the Newtonian coefficient of restitution. Such formulas are concerned largely with the inertia of the hammer and the pile, and any elastic deformations that occur during impact are accounted for by means of the coefficient of restitution. Other formulas have been derived on the assumption that the temporary elastic compression of the pile is the only energy loss that has to be included. Such formulas consider only the internal strain energy in the pile and pay no attention to its inertia. A few formulas have been published in which efforts have been made to combine the Newtonian energy loss with the temporary elastic compression losses in the pile, the soil and the driving cushion. However, such formulas are unsound from the point of view of theoretical mechanics because the effort to combine two theories of impact into one formula leads to a duplication of some of the energy losses.

B-5. Theory of Longitudinal Impact.—During recent years the phenomena of pile driving have been investigated on the basis of the theory of longitudinal impact of rods. This theory is concerned largely with the transmission of stresses within the pile. From such investigations as have been made to date, it appears as if the theory of longitudinal impact is more closely related to pile driving than are the theories on which most of the older formulas are based.

B-6. Reliability.—Some of the existing dynamic formulas are relatively simple, whereas others are quite complicated. It should not be assumed that the number of terms in the formula is a measure of its accuracy. All dynamic pile-driving formulas are subject to definite limitations and any dynamic pile-driving formula is nothing more than a yardstick to help the engineer secure reasonably safe and uniform results over the entire job. The use of a complicated formula is not recommended since such formulas have no greater claim to accuracy than the more simple ones.

B-7. Limitation of Driving.—For practical reasons it is necessary to have some means of deciding when to stop driving a pile. This determination is usually based on a dynamic formula into which numerical values are substituted. When this is done certain important facts must be kept in mind:

(*a*) The determination of bearing capacity by means of a dynamic formula refers only to a single pile. It is therefore necessary to keep in mind the probable subsequent behavior of the entire pile group.

(*b*) The resistance or bearing capacity determined by the formula is the dynamic resistance. The usual purpose for which the pile is driven is to support a static load. Therefore, it is necessary to keep in mind the probable relationship between dynamic resistance and static bearing capacity.

B-8. Static Formulas; Reasons for Use.—Because of the uncertainty that exists as to the relationship between the dynamic driving resistance and the subsequent static carrying capacity of the pile, formulas have been developed for determining the bearing capacity of the pile on the basis of purely static considerations. Most of the available static formulas are based on the classical earth pressure theories and some of them are rather elaborate.

B-9.—In order to use these static formulas, it is necessary to know the shape and the size of the pile and the physical properties of the surrounding soil. The necessary information about the pile itself is always available. The determination of the physical properties of the surrounding soil is a difficult problem as the driving of the pile often produces changes in the physical characteristics of the soil. It is necessary, therefore, to know the soil characteristics after the pile is in place in order to determine its bearing capacity by means of a static formula.

B-10.—The use of either a dynamic or a static formula must be accompanied by a considerable amount of experience and good judgment. Whenever a dynamic formula involving numerous variables is to be used, field measurements of all of the variables should be made in preference to office calculations of these quantities. The temporary elastic compression of the pile should be measured in the field and not calculated by means of static strain-energy formulas. The stiffness of the cushion block should be determined by experiment and not by assumption. The energy that is being delivered by the hammer should be checked in the field and not selected from tabulations in manufacturers' catalogs. Whenever a static formula is to be used, the physical properties of the soil should be determined in the laboratory and consideration should be given to the fact that the installation of the pile usually has some effect on the physical properties of the soil.

B-11. Bibliography for Report B, on Pile Formulas.—The following references apply to dynamic formulas:

(1) **"Piles and Pile Foundations,"** by J. S. Crandall, *Journal*, Boston Soc. of Civ. Engrs., May, 1931, Vol. XVIII, No. 5, p. 176.
(2) **"Dynamic Pile Driving Formulas,"** by A. E. Cummings, *Journal*, Boston Soc. of Civ. Engrs., January, 1940, Vol. XXVII, No. 1, p. 6.
(3) **"An Investigation of the Stresses in Reinforced Concrete Piles During Driving,"** by W. H. Glanville, G. Grime, E. N. Fox, and W. W. Davies, *Technical Paper No. 20*, Great Britain Scientific and Industrial Research Dept., Building Research Board, London, 1938.
(4) **"Das Problem der Pfahlbelastung,"** by O. Stern, published by William Ernst and Sohn, Berlin, 1908.

The following references apply to static formulas:

(5) **"The Ultimate Load on Pile Foundations: A Static Theory,"** by John H. Griffith, *Transactions*, Am. Soc. C. E., Volume LXX, December, 1910, p. 412.
(6) **"Die Tragfähigkeit der Pfahle,"** by Heinrich Dörr, Berlin, 1922.
(7) **"Pieux et Sonnettes,"** by E. Nöe and L. Troch, published by Gauthier-Villars, Paris, France, 1920.

B-12.—Load testing of piles is the only reliable method for determining the load which a pile can safely carry in relation to the shearing strength of the soil surrounding and underlying the pile. If such tests are carried to failure they measure directly the ultimate bearing capacity of a pile, as compared with pile formulas which are supposed to yield this value indirectly.

B-13.—A test pile is one on which a known load is placed in order to determine its load-bearing capacity. The purpose of a test may be (1) to guide the engineer in the selection of a proper type of pile and its load-bearing capacity, or (2) to confirm his assumptions in a design already prepared and to check the quality of the work. In the first case the test piles are specially driven and tested well in advance of the actual construction in order to serve as a basis for the design. In the second case the engineer, having already selected the type of pile and the assumed design load, selects certain piles for loading test purposes from those driven on the actual construction work. The test piles in this case are in essence a check on the design and on the builder's work. When unexpectedly poor results are found they constitute the reason for re-design or rejection of work installed. Whenever a pile is driven with the expectation that it may be subjected to a load test, it is essential that a complete record be kept of all data respecting the pile and the driving operation, particularly a detailed record of the driving resistance.

B-14. Method of Loading.—A suitable platform supported on the test pile or piles may be loaded with pig iron or other measured weights, or, where existing structures of sufficient size are conveniently located, jacking against such dead weight may prove to be more economical. If the jacking reaction is against a beam anchored to two neighboring piles, these piles should each be at least 10 ft from the pile under test. Hydraulic jacks, properly calibrated, give accurate measurement of the loads applied. The load should be applied in suitable increments (5 tons, for example); each increment should be allowed to rest for a specified period (as one hour). When the proposed design load is reached, that load should be maintained for a period of at least 24 hr. Then the load should be increased in increments to twice the proposed design load, or to failure. The elastic rebound should be observed by removing the load in similar steps as the loading operation.

B-15.—Allowable load on a pile as determined by a load test is frequently based on the simple rule that the total net settlement, after deduction of the rebound, shall not be more than 0.01 in. for each ton of load. In addition, one should require that the allowable load shall not exceed one half of the load at failure, or the maximum load reached during the test.

Respectfully submitted,
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April 20, 1941

* Not a member of the Society.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THEORY OF ELASTIC STABILITY APPLIED TO STRUCTURAL DESIGN

Discussion

BY LEON S. MOISSEIFF AND FREDERICK LIENHARD, MEMBERS, AM. SOC. C. E.

LEON S. MOISSEIFF,[24] AND FREDERICK LIENHARD,[25] MEMBERS, AM. SOC. C. E. (by letter).[25a]—In the limited space available for this paper, the writers had to confine themselves to the simpler and less differentiated problems. Mr. Balog has added valuable information by bringing forth a more complex, though not rare, case.

He treats the buckling case which involves not only the simultaneous action of bending and shear but, in addition, that of axial stress. As is to be expected, the coefficient of stability k will be smaller for bending, shear, and compression stresses acting together than for bending and shear only. In Table 12 he gives the greater reduction in the values of k as the compressive stress increasingly extends over the larger part of the section. Mr. Hill likewise calls attention to the combined uniform compression and shear.

Mr. Balog also has contributed a well-reasoned presentation of a method of deriving from the elasticity modulus variation, as determined from tension stress-strain curves, the modulus variation for compression. This procedure, if verified experimentally, promises a direct and valuable approach to the problem of critical stability. The curves of the strain scales in Fig. 10, when read vertically downward, from tension to compression, leave a memorable impression of the character of the metals considered. This is especially important when one realizes that they represent by far the most important structural metals.

The writers fully agree with Mr. Hill that experimental tests are needed to verify the theoretical results derived; in the introduction to the paper they have called attention to the need of comprehensive tests. This is especially

NOTE.—This paper by Leon S. Moisseiff and Frederick Lienhard, Members, Am. Soc. C. E., was published in January, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1940, by Louis Balog, Esq.; February, 1941, by Joseph S. Newell, Esq.; and March, 1941, by Messrs. Harold D. Hussey, H. N. Hill, and F. H. Frankland.

[24] Cons. Engr., New York, N. Y.

[25] Engr., Leon S. Moisseiff, New York, N. Y.

[25a] Received by the Secretary April 18, 1941.

true of Eq. 31, which gives the size of transverse stiffeners of column plates already stiffened by longitudinal stiffeners. The writers have not been able to find a better method to determine the size and spacing of stiffeners in explicit form. It may be pointed out, however, that stiffeners which were designed by Eq. 31 were checked numerically by the method proposed by Mr. Seydel (29).[25b] The results were satisfactory.

Mr. Hill also calls attention to the partly clamped edge condition assumed by the writers in the case of shear in panels having a length-to-width ratio of less than 0.5. He points out that, by transposing the length and width terms, the ratio becomes 2 and thus the assumption of partly clamped edges is not obvious. The writers were particularly concerned with the design of plate girders. For this reason the length-to-width ratios were taken less than unity as is common in such structures. The width was always intended to be the distance between toes of flange angles, and the length the distance between vertical stiffeners. Shear stiffeners, when closely spaced, tend to increase the buckling stability in the compression zone. This was not taken into account in the fundamental equation, but was compensated for by assuming partly clamped edge conditions for length-to-width ratios of less than 0.5.

The increase in k for partly clamped edges, as assumed by the writers, over that of free edges is not substantial. For a length-to-width ratio of 0.40, the increase is 2.7%; it increases to only 24% for a ratio of 0.20.

Mr. Hill also points out, with justification, that the modulus factor at the yield point should not be taken equal to zero for aluminum alloy 27S-T. It is correct, as stated by him, that for the yield strength of this alloy the modulus factor is approximately 0.4. The effect of this is, no doubt, of importance for airplane design but is of no consequence in the field of heavy structures where much higher factors of safety are used.

He also makes a valuable contribution by suggesting expressions that would make Eqs. 21 and 22 applicable to any number of stiffeners and not only to three, as limited by the writers.

He suggests different formulas for spacing of stiffeners—Eqs. 57 and 58. These equations are generally more applicable for a greater range of length-to-width ratios. The writers wish to point out, however, that their formulas are considerably simpler and suitable for practical design of plate girders. For plates, where the stiffeners can be placed so that the length-to-width ratio is considerably greater than unity, the formulas of the writers do not apply. They suggest that the equations derived by Mr. Hill be used.

Mr. Hill further discusses the twisting stability of beams. The writers are well aware of this phenomenon and its great importance in structural design. Space limitations, however, precluded a discussion of the problem. The buckling stability of compression flanges, including torsional rigidity, has been treated elsewhere.[26]

[25b] Numerals in parentheses, thus (29), refer to corresponding items in the Bibliography in the Appendix of the paper.

[26] "Design Specifications for Bridges and Structures for Aluminum 27S-T," by Leon S. Moisseiff, pp. 2 and 3, and 59 to 66.

Mr. Hussey points out that, for values of $\frac{d}{t}$ for outstanding parts of columns, the minimum value of k was used in computing Table 4(*b*) in order to allow for initial curvature. He is of the opinion that it would be more reasonable to assume a value of k equal to twice the value assumed, which would result in increasing the values of $\frac{d}{t}$ by 41%. The writers believe that this would be too optimistic an assumption and prefer to adhere to the original values until careful tests justify an increase in the width-to-thickness ratio.

Mr. Hussey also calls attention to the fact that, for stresses less than the allowable, $\frac{d}{t}$ should be increased in direct proportion to the ratio of allowable to actual stress. This has been done in the paper.

As bridge and structural engineers, the writers had in mind the rational and economical design of structures intended to carry substantial loads and which themselves weigh many tons. They did not take into consideration the design of airplanes and similar structures where the dead weight is of the utmost importance and for which the limits of safety must be stretched to the utmost. Professor Newell correctly has called attention to the neglect of the writers to point out the residual strength left in many members after buckling has become evident. In the mind of the bridge engineer, structures in which elements have wrinkled into visible waves have lost their practical usefulness, as Professor Newell has well stated. Preoccupation with their field made the writers forget to limit properly the last sentence in the "Synopsis."

Mr. Frankland's comment is of special value because it comes from one so thoroughly familiar with the ever-active development of steel structures and the increasing demand for their improvement from the viewpoints of the designer as well as the fabricator.

The aim of the writers in presenting the paper was to call the attention of designers of engineering structures to the fact that the rules for the stability of the component parts of girders and columns, as given in most specifications, are crude and simple, and that "the very simplicity of the rule confines the application to narrow limits and hampers the design and its economy." They attempted to gather from the vast literature on elasticity, which has become available since about 1910, the most essential results relating to the stability behavior of metals, and to deduce from them formulas which would give the designer ample freedom to utilize the material in the most economical manner and which, at the same time, could be applied readily.

The writers feel obliged to the participants in the discussion for the valuable criticisms and contributions made to the paper, thereby adding to its value. The interest that this rather involved paper has evoked in engineers engaged in the design of metallic structures may serve as a sign of its timeliness. It is gratifying to note that comprehensive tests on elastic stability problems are at present being considered by interested professional bodies.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

CAVITATION IN OUTLET CONDUITS OF HIGH DAMS

Discussion

By Hunter Rouse, Assoc. M. Am. Soc. C. E.

Hunter Rouse,[22] Assoc. M. Am. Soc. C. E. (by letter).[22a]—Cavitation research has progressed to a comparatively refined stage in several of the engineering professions, but Messrs. Thomas and Schuleen appear to be the first to have investigated, by laboratory experiments, the occurrence of cavitation in masonry structures. Although it might be argued that the phenomenon is the same regardless of the surroundings in which it occurs, the writer feels that much credit is due the authors, both for the development of the experimental technique which they describe and for calling the attention of the civil engineer so forcibly to the dangers of poor hydraulic design.

While the writer therefore voices only commendation for the major part of the paper under discussion, he believes that the authors' comments on the "Nature and Causes of Cavitation" leave much to be desired. The attempted proof of the existence of enormous local intensities of pressure seems particularly subject to criticism, owing to the artificial (if not physically impossible) nature of the assumption of constant kinetic energy in the specimen slug of water. Perhaps quite as artificial, although considerably more elegant, would have been the assumption of a momentary spherical bubble of water vapor within an initially stagnant, incompressible liquid of arbitrary volume; the work done by external pressure in accelerating the liquid toward this center would then yield the ultimate kinetic energy as the bubble collapsed, leading in a more reasonable manner to the same conclusion of an infinite local velocity and an infinite local pressure intensity at the point of collapse. Indeed, one need only hear the violent shocks which are produced by discharging steam under water to concede the issue that the collapse of such vapor pockets might well lead to exceedingly high local stresses. Of particular interest, in the adap-

Note.—This paper by Harold A. Thomas, M. Am. Soc. C. E., and Emil P. Schuleen, Assoc. M. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jerome Fee, Assoc. M. Am. Soc. C. E.; and April, 1941, by Messrs. V. E. Leman, P. S. O'Shaughnessy and E. S. Randolph, and Carroll F. Merriam.

[22] Prof., Fluid Mechanics, State Univ. of Iowa; Cons. Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[22a] Received by the Secretary April 2, 1941.

tation of this approach to the case of a compressible liquid, is the fact that a two-dimensional or three-dimensional positive pressure wave must invariably be followed by a negative wave of similar order of magnitude.[23] Such conditions would seem to point even more strongly to the stress-reversal theory of boundary failure.

On the other hand, high-speed motion pictures by H. Mueller[24] of the collapse of vapor bubbles produced in cavitation would indicate that the bubbles tend to flatten on the downstream side rather than to dwindle concentrically. In other words, the impact area is then of finite magnitude rather than infinitesimal, and the computed terminal velocities are therefore finite even if the fluid is assumed incompressible. Conclusions of this nature would tend to invalidate much of the authors' basic hypothesis, as do piezoelectric measurements by J. Ackeret[25] of only moderately high instantaneous stresses in zones of cavitation. Moreover, the effect of pitting has been noted in Pelton-wheel buckets, where there is no reason to expect the formation of vapor pockets, and the method of passing test specimens repeatedly through free jets of water has been used by Professor Ackeret in comparing the resistance to pitting of various materials.

Although the authors' assumption of unbelievably high intensities of pressure has long been a favorite explanation of the pitting effect, it is possible that the theory of fatigue through repeated impact of not-too-great magnitude will eventually be more generally accepted. In other words, whereas vaporization of the liquid is the essential feature of cavitation itself, the existence of the vapor state apparently is not essential, but merely contributory, to the phenomenon of pitting through liquid impact. The term "impact," however, is so often used erroneously in connection with liquid motion that its true import has become obscured. For instance, a continuous jet of liquid deflected by a vane is frequently cited in hydraulics to illustrate the "impact" of a liquid upon a solid boundary, and the stagnation pressure at the point of zero velocity is even more frequently called the "impact pressure." This is in no way related to the case of impact dealt with in theoretical or applied mechanics, for no discontinuity or unsteadiness of the liquid is involved. It is not surprising, therefore, that such a fictitious case of impact—the steady deflection of a continuous liquid jet—produces no sign of pitting in the solid boundary. On the other hand, if the free jet is not continuous, but composed of drops or even slugs of liquid, pitting of the solid by true liquid impact is readily obtained.[26]

Nevertheless, it is difficult for the hydraulic engineer to conceive of liquid impact as having essentially the same capacity for destruction as the impact of one solid upon another. Yet this was simply demonstrated by H. Föttinger[27] through the use of a glass tube partly filled with liquid, evacuated,

[23] "Hydrodynamics," by H. Lamb, Cambridge University Press, 6th Ed., 1932, pp. 491, 492, and 525-527.

[24] "Hydromechanische Probleme des Schiffsantriebs," edited by G. Kempf and E. Foerster, Hamburg, 1932, pp. 311-314.

[25] *Loc. cit.*, p. 239.

[26] *Loc. cit.*, p. 238.

[27] *Loc. cit.*, pp. 251-254.

and sealed at both ends. By inverting the tube, the slug of liquid within was made to drop to the bottom, producing a shock akin to that of a metal rod falling the same distance; indeed, the shock was once so great as to break the lower end of the tube, atmospheric pressure then driving the liquid upward with such force as to break the upper end as well. Needless to say, air left within such a tube would tend to cushion the blow, just as the presence of air in any zone of liquid or solid impact tends to reduce the destructive effect of repeated momentary stresses.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MAXIMUM PROBABLE FLOODS ON PENNSYLVANIA STREAMS

Discussion

BY WALDO E. SMITH, M. AM. SOC. C. E.

WALDO E. SMITH,[21] M. AM. SOC. C. E. (by letter).[21a]—Inasmuch as the author has divided his comprehensive paper into two main parts, the writer will attempt to do likewise with his discussion.

With reference, first, to the part of the paper on flood-producing storms, Mr. Ruff has made an extensive survey of the location, duration, and extent of storms in the development of profiles along two axes, all of which work was done with the ultimate end of application to Pennsylvania. With regard to the Ohio Axis it would seem that, for general application, this should have been curved southward, conforming in the main to the Ohio-Mississippi channel with a Mississippi Axis continuing northward to which the Missouri and Iowa storms should be applied for profiles along that axis. This is in line with the author's statement that distances along the profile represent approximate distances from the normal sources of precipitable moisture. A sequence of isentropic charts which show the movement of tongues of moist, warm air inland from the Gulf of Mexico indicates that there is little justification for drawing the lower part of the Ohio Axis, as shown in Figs. 3 and 4. The effect of the selection of the position of the axes on the profile derived from the storms is shown in Fig. 22, in which the writer has replotted, with respect to the axes shown, the data in Table 3(*b*) for 5-day storms on 6,000 sq miles, holding the position of station 12 and the eastward part of the Ohio Axis unchanged. The profile drawn for the Mississippi-Ohio Axis is comparable to the one in Fig. 5 and the corresponding one in Fig. 3. The author's profile from those figures is shown by the dotted line. The differences to the right of mile 870 are not due to position of axis, but rather to interpretation of data. It is acknowledged that the writer's profile has not benefited by comparison with profiles for shorter

NOTE.—This paper by Charles F. Ruff, M. Am. Soc. C. E., was published in September, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Messrs. Joseph L. Benson, H. Alden Foster, and Edgar E. Foster; January, 1941, by C. S. Jarvis, M. Am. Soc. C. E.; and February, 1941, by Messrs. Gordon R. Williams, and Emil P. Schuleen.

[21] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[21a] Received by the Secretary March 28, 1941.

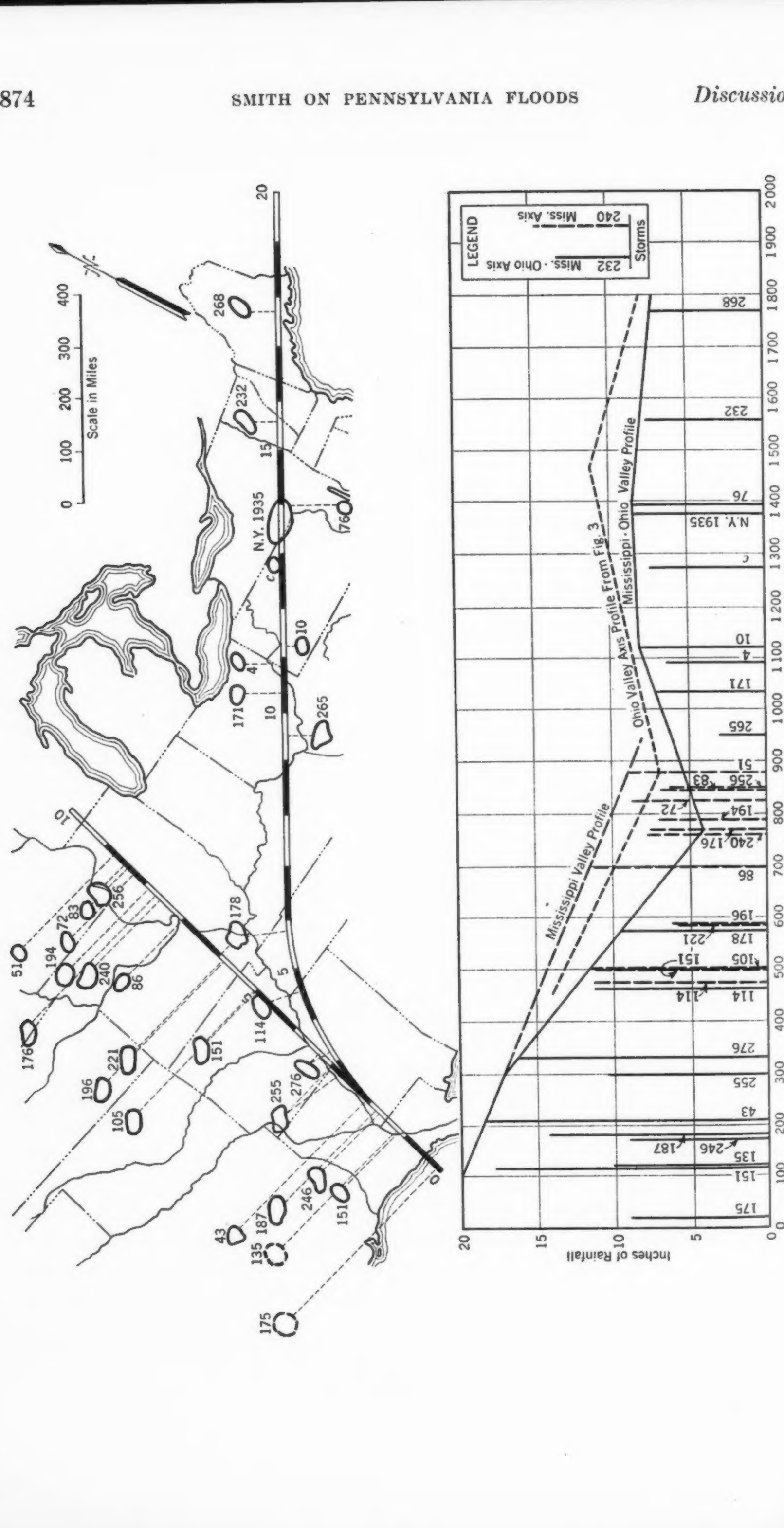

FIG. 22.—FIVE-DAY STORM RAINFALL ALONG THE MISSISSIPPI-OHIO VALLEY AND MISSISSIPPI VALLEY AXES IN SUMMER, ON 6,000-SQ MILE AREA

storms and smaller areas. The exact positions of storms 135 and 175 are not known, inasmuch as they were not shown in Fig. 3.

Table 3(*b*) does not include certain important storms in the critical region from stations 8 to 10 along the Ohio Axis which might, on investigation, serve to raise the profiles in that vicinity and serve further to alter the entire series of profiles for the summer condition. These are, most notably, the storms over Ohio in July 1913,[22] August 1935,[23] and June 1937.[24] One other of later origin than those studied by the author, which would serve to raise sections of the profile referred to, occurred in eastern Kentucky on the night of July 4–5, 1939.[25] Although the flood-producing portions of all of these rains occurred in periods of from 12 to 24 hr, they would serve to cause adjustments of the nature referred to by the author under "Estimate of Maximum Probable Daily Rainfall: Atlantic Summer Storms," for the longer periods of time. There were some antecedent or succeeding rains that might increase 4-day and 5-day values above those shown.

In the discussion of data for maximum probable rainfall for short durations, it is apparent that the author must have prepared profiles for 1, 100, and 3,000 sq mile areas also, as indicated by the data in Fig. 7 for station 12. In Figs. 7(*b*) and 7(*c*) data are given for these areas, the inspection of which shows that the points indicated by small circles in Fig. 7(*a*) are for 1 sq mile. Many of the points shown by small dots in this figure are quite far afield from station 12 on the Ohio Axis but they were apparently considered as directly applicable to that station as indicated by their use. A local storm at Cambridge, Ohio (probably No. 41 in Fig. 7(*a*)), resulted in 7.09 in. of rain in 90 min. The gage was presumably very near the eye of the storm, whereas the entire area receiving ½ in. or more of precipitation was apparently only 25 or 30 sq miles.[26] Storm No. 44, which occurred on June 3, 1921, at Cincinnati, is an unofficial record that has been quite generally accepted.[27] Weather Bureau records of that time make no mention of an unusual rain and the records of three official stations in the Cincinnati vicinity show amounts of only 0.51 in., 0.34 in., and 0.05 in., indicating that the extent of the storm must have been small. The nature of these two storms suggests that there should be a wider separation of the 1-sq mile storm from the 100-sq mile storm for summer conditions than is shown by Fig. 7(*b*), especially for short periods of time. The uppermost curve for 1 sq mile should probably be substantially higher than is shown. This idea is further supported by the fact that, with spotty rainfall, it is rarely possible to measure the maximum rainfall with the wide scattering of rain gages that have been prevalent. This has been shown by C. W. Thornthwaite,[28] and is

[22] "The Ohio Water Problem," by the late C. E. Sherman, M. Am. Soc. C. E., Ohio State Univ., 1915, p. 10.

[23] "Flood of August 1935 in the Muskingum River Basin," *Water Supply Paper No. 869*, U. S. Geological Survey.

[24] "Climatological Data," U. S. Weather Bureau, Ohio Section, June, 1937, Bucyrus and other stations; also unpublished maps and other records, Climatological and Physiographic Div., SCS.

[25] Unpublished Report by the U. S. Engr. Office, Cincinnati District, Cincinnati, Ohio.

[26] "The Ohio Water Problem," by the late C. E. Sherman, Ohio State Univ., 1915, p. 16.

[27] "Cloudburst Phenomena," unpublished report by Ivan E. Houk, M. Am. Soc. C. E., presented before the Colorado Section, Am. Soc. C. E., June, 1925.

[28] "The Reliability of Rainfall Intensity Frequency Determinations," by C. Warren Thornthwaite, *Transactions*, Am. Geophysical Union, 1937, pp. 476–484.

further supported by inspection of isohyetal maps[29] of intense summer storms of the Muskingum Climatic Research Project of the U. S. Soil Conservation Service. Very small centers of 5 in. or more in 4 to 6 hr are quite common, whereas areal averages of these amounts on 100 sq miles require longer periods of rainfall and are very few. Fig. 18 indicates that rainfall data from widely scattered stations alone give insufficient rainfall on small watersheds to account for the excessive flows sometimes observed, lending further support to Thornthwaite's contention. For areas in excess of 100 sq miles, the determined values of runoff seem to be reasonably consistent with the observed data, and the author is justified therefore in placing that value as the lower areal limit on his studies. With reference to large areas, and in line with Mr. Ruff's comment on this part of the work, the writer believes that probably an average of 2 in. of rainfall on 6,000 sq miles in 4 hr in summer is just about as rare an occurrence as in winter and that the left-hand ends of the lower summer curves of Fig. 7(*b*) should dip downward more sharply. In other words, there should probably be a much wider general divergence of the left-hand ends of the summer curves than is shown.

The author has used the term "inches of rainfall" frequently throughout his text and on figures without indicating whether this is "inches or more" as is used in various studies, or an average value. It has been interpreted by the writer as being the latter.

Reference is made to the term "duration in days" which the writer has presumed to mean the number of consecutive days on which the rainfall indicated was recorded, rather than actual duration, which for a recorded 2-day rain may have been a period varying from a very short time to 48 hr with an approximate average of about 24 hr; and for a 3-day record of rain may have been in a period ranging from a little more than 24 hr to 72 hr with 48 hr as the approximate average, as shown by C. W. Sherman[30] and G. A. Hathaway,[31] Members, Am. Soc. C. E.

With reference to the second part of the paper on standard floods, the author has indeed developed an interesting and comprehensive method that would seem to give consistent results for a given region. Area-correction and shape-correction factors are especially well treated. He has been wise to counsel against the use of this method for design purposes, without taking proper account of the numerous other factors involved that do not appear in their proper weight in such an analysis.

The writer has found that lag time from the center of mass of the rainfall producing runoff to the center of mass of the runoff, as suggested by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt,[32] Assoc. M. Am. Soc. C. E., is generally a more useful term than the crest lag used by the author; and it is

[29] An example is found in "Reliability of Station-Year Rainfall-Frequency Determinations," by Katharine Clarke-Hafstad, *Proceedings*, Am. Soc. C. E., November, 1940, p. 1608.

[30] "Actual Duration of 'One-Day' and 'Two-Day' Rain Storms," by Charles W. Sherman, *Civil Engineering*, March, 1939, p. 179.

[31] "Estimating Maximum Flood Flow as a Basis for the Design of Protective Works," by Gail A. Hathaway, *Transactions*, Am. Geophysical Union, 1939, pp. 195–203.

[32] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

less subject to variation on a given watershed for various amounts of rainfall, within a given time unit, than the crest lag.

The writer frequently has found that, for many purposes, the data given in Fig. 12 showing individual values for the distribution graph may be more conveniently plotted as a summation graph as shown in Fig. 23. The particular advantage of this form is that one can always select values the first time that total 100%, although the presentation as given in Fig. 12 has the advantage that the peak period is always obvious.

Under the heading "Development of the Standard Floods: The Standard Watershed," Mr. Ruff states: "The graphs would be equally applicable to any watershed having the same lag." This sentence should have been qualified in accordance with practice elsewhere, with reference to its application to the Appalachian highlands.

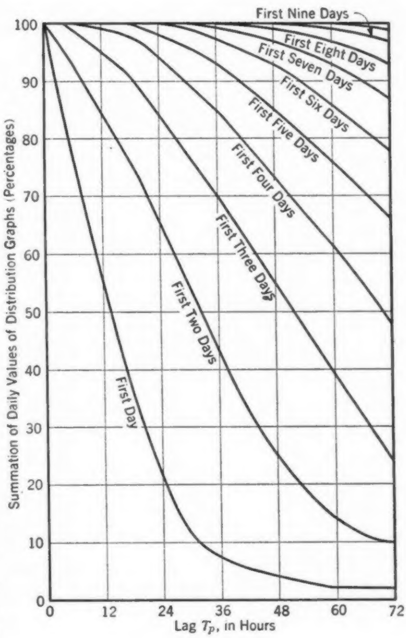

FIG. 23

Again, under the heading "Development of the Standard Floods: The Standard Flood," the statement is made that:

> "The standard floods vary closely as the 0.3 power of the watershed area for areas between 100 and 1,000 sq miles, and the total peak flow varies as the 0.7 power of the area. From 1,000 to 10,000 sq miles, the Atlantic summer flood continues to vary at this rate, but the winter floods vary more nearly as the 0.4 power, and the Ohio summer flood as the 0.45 power of the watershed area."

In other words, the crests of standard floods (in cubic feet per second per square mile) vary inversely as the 0.3 power of the watershed area for areas between 100 and 1,000 sq miles, or the maximum probable flood in terms of cubic feet per second varies as the 0.7 power of the area. From 1,000 to 10,000 sq miles the Atlantic summer flood continues to vary at this rate, but for the winter floods, the peak per cubic foot per second per square mile values vary more nearly as the inverse 0.4 power, and the Ohio summer floods as the inverse 0.45 power of watershed areas.

It seems undesirable that after all the care that the author has taken to arrive at rational values, it is then necessary to seek an arbitrary runoff coefficient to apply to the results in order to arrive at the desired answer. Still, the writer has no alternative to offer for winter storms. However, the factor 120% impresses the writer as being small. The runoff from winter storms on bare ground in the northern Appalachian highland region frequently approaches 100% for large storms. The 20% then, roughly, is the allowance for melting snow and ice. On an area on which 5 in. of rain in 24 hr produces the maximum winter flood this allowance is only 1 in. If the March, 1936, flood on the Pennsylvania headwaters of the Ohio River had not been preceded by a mild period a few days earlier which greatly reduced the snow cover, the resulting flood stages would have been much higher. In Fig. 5 the rainfall causing the March, 1936, flood is shown for a 6,000 sq mile area as having a value of about 7 in., whereas the profile indicates about $8\frac{1}{2}$ in. Thus, for this particular area and storm, and for a 5-day duration, there would be an allowance of $1\frac{1}{2}$ in. plus 20% of $8\frac{1}{2}$ in. for the melting snow cover, which seems adequate; but for smaller areas or other durations the question still remains.

With regard to summer floods, the 90% factor seems too high. One might find a new approach to this value through an application of infiltration values, much data on which are now becoming available in Pennsylvania. Before leaving the subject of runoff coefficient the writer believes that, fortunately, the author has twice refuted his opening general statement that imperviousness is the same for every flood on a watershed. He first refutes this idea in the second sentence following its statement when he declares that "The condition of the ground at the time the storm occurs * * * is also a factor," and again by inference in the methods of analysis used for the modification of runoff coefficient because of season of the year. It would be well for hydrologists and hydraulic engineers to cease considering degree of imperviousness as a watershed constant, inasmuch as overwhelming evidence shows that it varies not only from season to season, and storm to storm, but within a storm period.

Corrections for *Transactions:* In September, 1940, *Proceedings*, Fig. 1, Storm No. 248 should be Storm No. 249; Fig. 3, delete Storms Nos. 51, 86, and 194, and add Storm No. 242 from Fig. 4; in Fig. 4, delete Storm No. 242, add Storms Nos. 51, 86, and 194 from Fig. 3, and change Storm No. 157 to Storm No. 151; in Fig. 5, Storms Nos. 51, 86, and 194 should appear as broken lines; in the caption to Fig. 5 change "4-Day Storm" to "5-Day Storm"; in Table 3(*b*), column of distances along the axis, change Designation N Y 1935 from 1,450 to 1,350 and Designation 76 from 1,495 to 1,395; on page 1254, line 25, change "1924" to "1921"; on page 1267, line 17, change "(see Appendix)" to "(see Items (7), (8), (9), and (10), Appendix)"; on page 1273, line 13, change "(see Appendix)" to "(see Appendix, Items (13) to (16))"; on page 1275, add to Items 9 and 10 "(unpublished)."

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DISCUSSIONS

RECOMMENDED PRACTICE AND STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

Discussion

BY MESSRS. EGIDIO O. DI GENOVA, A. J. BOASE, WILLIAM RICHARD WALLIS, AND ELWYN E. SEELYE

EGIDIO O. DI GENOVA,[73] ASSOC. M. AM. SOC. C. E. (by letter).[73a]—In computing the effective section of a spirally reinforced concrete column, the shell (or a part of the shell) has been disregarded for a long time. In the Joint Committee Report (Section 851(*b*)) the shell is considered not only as a protection of the spiral, but also as an integral part of the entire column section.

Formerly, this part of the section was ignored.[74] Actually, it was an allowance made to compensate for the eventual cracking of the outside shell because of overstress, plastic flow, faulty aggregates, imperfect mixing, etc. The 1940 Report permits the designer to ignore these possibilities. The full section and the "effective section" are now identical, which implies that, when the shell becomes cracked, the spiral will act to resist the load that otherwise would have been assigned to the center core and its vertical reinforcement. Experiments reported by the Joint Committee are said to have given satisfactory results to justify this newly adopted method.

Is this new theory to be applied in all cases; or is it limited to cases in which the resultant diagram of combined bending and concentric load stresses produces a totally compressed column section?

Columns completely in compression occur in common types of construction because in many instances the tension effect produced by bending is nullified, thus indicating that the spiral reinforcement is active. Consider two of the most common cases that may occur in the daily practice of an engineer:

NOTE.—This Report was published in June, 1940, *Proceedings*, Part 2. Discussion on this Report has appeared in *Proceedings*, as follows: September, 1940, by L. J. Mensch, M. Am. Soc. C. E.; November, 1940, by Messrs. John C. Sprague, and Walter R. Hnot; December, 1940, by Edward C. Gould, Assoc. M. Am. Soc. C. E.; February, 1941, by O. G. Julian, M. Am. Soc. C. E.; March, 1941, by Jacob Feld, M. Am. Soc. C. E.; and April, 1941, by Messrs. Harold E. Wessman, N. T. Stadtfeld, C. A. Ellis, R. H. Sherlock, S. C. Hollister, Thomas K. A. Hendrick, Morris Berman, F. R. McMillan, and Meyer Hirschthal.

[73] Asst. Engr., Board of Water Supply, New York, N. Y.

[73a] Received by the Secretary March 17, 1941.

[74] *Proceedings*, Am. Soc. C. E., October, 1924, Eq. 42, Section 162, p. 1203.

(1) Columns that resist extremely large flexural moment compared with the imposed axial load (that is, a relatively small load applied an unusual distance from the axis); and

(2) Exterior columns at the top floor of a rigid-frame structure. In the latter case a formal analysis may reveal an unusually large induced moment at the top of the column, although the vertical load is relatively small. If, in addition, a wind is blowing against a projecting ornamental bracket, superposing another moment with the same sign as the foregoing, the combination will demonstrate, roughly, the ideal case of a vertical member subject to an appreciable bending moment and a relatively negligible axial load. Such a column may be considered as a beam subject to bending action.

In such a member, spiral reinforcement helps to resist any diagonal tension. The position of the neutral axis is well defined and is independent of the spiral reinforcement. If the elongation on the tension side and the shortening on the compression side are assumed to vary linearly (that is, plane sections remain plain after bending), the spiral will have a marked influence in producing cracks in the concrete beyond the core line. For this reason the spiral reinforcement cannot be considered, at present, as effective in cases (1) and (2) as in columns under complete compression. In adopting the new spiral column theory, it would be better to make a distinction between types of columns. In cases (1) and (2), discussed herein, the writer would recommend adhering to the 1924 Report, and exclude the shell from the section area. He would prefer the eccentric load criteria in design, which is most nearly compatible with modern design practice.

Although he appreciates the value of the reserve strength afforded by the spiral reinforcement, the writer is not certain that tests reported in defense of this principle include cases (1) and (2). Spiral columns are used for heavy loads and, although cases (1) and (2) are exaggerated, some concrete structure with spiral reinforcement may some day be subjected to a triangular stress variation with tension at one side. An investigation of such cases in the light of the 1940 Report of the Joint Committee would be profitable.

A. J. Boase,[75] M. Am. Soc. C. E.[75a]—Throughout the 1940 Report, the Joint Committee emphasizes repeatedly the careful consideration that was given to the subject of frame moments and to the omission of prescribed moment coefficients. In this respect the Report does not differ greatly from its predecessor. The 1924 Joint Committee Report gives coefficients for cases only of equal span and clearly states that for

> "Continuous beams with unequal spans, or with other than uniformly distributed loading, whether freely supported or restrained, shall be designed for the actual moments under the conditions of loading and restraint."[76]

[75] Mgr., Structural Bureau, Portland Cement Assn., Chicago, Ill.

[75a] Received by the Secretary March 17, 1941.

[76] *Proceedings*, Am. Soc. C. E., October, 1924, Section 111, p. 1191.

The omission of coefficients is not without precedent in building codes. For cases of unequal span, or with other than uniformly distributed loading, the 1928 American Concrete Institute Code[77] recognized the impossibility of prescribing moment coefficients that would produce a design efficiently utilizing the materials involved, or of maintaining a nearly uniform factor of safety throughout. This code states that the prescribed coefficients are permitted only for cases of "approximately equal span." No attempt was made to define "approximately equal." For beams or slabs of unequal span the code states:

> "Continuous beams with substantially unequal spans or with other than uniformly distributed loading, whether freely supported or restrained, shall be designed for the maximum moments resulting from the most severe probable combination of loading and restraint. Provision shall be made where necessary for negative moment near the center of short spans which are adjacent to long spans, and for the negative moment at the end supports, if restrained."

The 1936 American Concrete Institute Code, as well as the proposed 1941 standard, do define the word "approximately" by stating:[78]

> "For purposes of applying this method [prescribed coefficients] 'approximately' shall be considered to mean that the longer of two adjacent spans shall not exceed the shorter by more than 20 per cent * * *."

The use of the given coefficients is further restricted by limiting them to cases where the uniform live load does not exceed three times the uniform dead load. In all other cases the method to be followed is defined by one sentence—

> "All members shall be designed to resist at all sections the maximum bending moments and shears produced by dead load, live load and wind load, as determined by the principle of continuity."

After much study, the Joint Committee found it just as impossible, as had previous code-making groups, to give simple coefficients that ignore such important elements as ratio of live to dead load, concentrated loads, ratio of span lengths in adjacent bays, ratio of column stiffness to beam stiffness, and ratio of support width to span length.

The Committee also felt: That during the past ten years more easily applied methods of analysis had been made available; that these methods could be further simplified in most cases without undue sacrifice of accuracy; and that, with the prevalent use of higher strengths, lighter weights of concrete, and higher unit stresses, there was need for using the better methods of analysis. The Committee therefore incorporated sections 805(*a*), 805(*b*), and 805(*c*) into its Report.

Section 805(*c*) permits a designer wide latitude as to what "ordinary" building construction is and as to approximations that may be made. For any but the most unusual cases it was definitely in the minds of the Committee that satisfactory results would be obtained from such a scheme as is written in the 1936 American Concrete Institute Code,[79] whereby columns may be

[77] Joint Code, Building Regulations for Reinforced Concrete, Am. Concrete Inst., Chapter 7, Paragraph 710(a).

[78] A. C. I. Building Code, Building Regulations for Reinforced Concrete, Chapter 7, Paragraph 702.

[79] *Loc. cit.*, Paragraph 702(a)(c).

assumed fixed at the far ends from the floor under consideration, using a loading pattern as described in Section 803(*a*). For ordinary building frames it was felt that two cycles of moment distribution would give sufficient accuracy, and, for those unfamiliar with the process of moment distribution, a formula based on that method is given in Appendix 2.

The following statement appears in Section 859:

> "In building frames, particular attention should be given to cases of unbalanced floor loads on both exterior and interior columns and of eccentric loading due to other causes. Wall columns should be designed to resist moments produced by: (*a*) loads on all floors of the building, (*b*) loads on a single exterior bay at two adjacent floor levels, or (*c*) loads on a single exterior bay at one floor level."

This statement is especially directed at the "unusual" type of building where, in the judgment of the designing engineer, a rigorous analysis is necessary. When an ordinary building design is involved (to be consistent), it was thought that the direct load on all columns should be determined by loading all panels at all floors adjacent to the column. For an exterior column this direct stress should be combined with the column moment obtained by loading only the exterior bay of the floor adjacent to the column under consideration. The design moment for interior columns can be determined with sufficient accuracy by loading two adjacent bays and distributing the difference of the end moments to the columns above and below in proportion to their stiffness.

Some questions have been raised regarding Section 808(*b*). Possibly this section should have been more closely connected with Section 805.

Section 802, Assumption 4, and Section 804(*d*), define the area that is to be used in computing the moment of inertia of beams connecting columns. Some method is needed, however, for the design of intermediate floor beams. A provision was made for this in Section 808(*b*) as follows:

> "In the case of beams framing into a girder, the restraint due to the torsional resistance afforded by the girder may be assumed as equal to that which would be furnished by a column having a stiffness factor equal to one-half the average stiffness factor of the beams framing into the girder."

The Committee does not suggest that this approach is rational, but its studies indicated that this device seemed to give reasonable results. It was felt that a start in this direction should be made even though the torsional properties of a reinforced-concrete member are not well understood.

There has been some misunderstanding regarding Section 808(*d*), in which it is stated:

> "(*d*) The moment at the face of a support may be used for proportioning the member. This moment may be obtained approximately from the moment at the centerline by subtracting a quantity $V\,a/3$, where V is the shear at the face of the support and a is the width of the support."

After much discussion, the Committee decided on center-to-center distances rather than clear span in the determination of frame moments. When the analysis is first started, that is about all that is known. Probably there is little flexure of the beam after it joins the column, and "clear span" is probably somewhat closer to the truth. However, the stiffness and the carry-over factor are more difficult to determine, the analysis is more complicated, and in most cases there is little practical difference in the final results. With centerline moments, the unbalanced moments should be corrected for shear. This correction is equal to one half $V a$, but to be conservative the Committee used one third instead of one half. This assumes that the point of contraflexure will be shifted away from the support by locating the corrected moment at the face of the support and drawing through that point a curve parallel to the original moment curve.

It is recommended in the Report that wind stresses be taken into account in buildings where wind is a factor. However, no method has been recommended for making the wind analysis. It was felt that any of the current methods were applicable to concrete frames and no special recommendations were needed in this Report.

William Richard Wallis,[80] Assoc. M. Am. Soc. C. E.[80a]—The Joint Committee has not been afraid to state its mind in forming a recommended practice and standard specifications, and the members should be congratulated for their courage. Certainly, too, they have done some good work. The writer would like to add a few suggestions.

In Section 829 there should be a clause which prohibits simply stopping off bars in the tension face of a beam. It should be required that the bars be extended at least twelve diameters into a region of compression. This rule can be satisfied easily by bending the bar out of the tension zone and across the neutral axis, or by extending it past the inflection point. By so doing, the theoretical requirements are satisfied, and the bar can be relied upon to function at the section desired.

For walls resting on soils, Section 876(*b*) requires that the resultant pressure at the base fall at least within the middle third of the base. For a wall where no vibration is anticipated, this requirement is often unnecessary and adds considerably to the cost of the structure. Unequal settlement of the footing is often unimportant, and the appearance of the wall tilting can be avoided by giving the exposed face a slight batter.

In Section 877(*d*) the Committee has recommended that longitudinal reinforcement for shrinkage and temperature changes in retaining walls be provided near the exposed face to an amount of not less than 0.50 sq in. area per ft of height. For thin walls, this is a rather high percentage of the cross-sectional area of the wall. The writer would like to ask the Committee for some discussion on this important subject of shrinkage and temperature reinforcement.

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[80] Structural and Civ. Engr., Allied Engrs., Inc., Fleet Operating Base, Terminal Island, Long Beach, Calif.

[80a] Received by the Secretary March 26, 1941.

ELWYN E. SEELYE,[81] M. AM. SOC. C. E. (by letter).[81a]—Referring to Sections 809(*c*) and 510, with particular reference to slabs bearing on masonry walls and to building expansion joints, the writer has provided the following insurance against cracking:

(*a*) Dowels were placed in the slab extending through the continuous flashing into the wall above to provide bond between the slab and the wall;

(*b*) Additional resistance to shear between the slab and the wall was provided by corner columns in the two upper stories;

(*c*) A small marginal beam was brought around the entire perimeter of the slab (this beam was reinforced with six $\frac{1}{2}$-in. round reinforcing bars for the purpose of preventing the development of vertical cracks); and to prevent corner at curling of slab.

(*d*) These buildings were cut into maximum lengths of about 150 ft by means of expansion joints.

The general purpose of the foregoing precautions was to provide against relative thermal expansion, which is believed to be the main cause of the cracks in other buildings. The large U. S. Housing Administration, known as Queensbridge, situated in the Borough of Queens, New York City, constructed in 1939, showed absolutely no cracking after two years.

Correction for *Transactions:* In March, 1941, *Proceedings*, p. 458, line 8, change "the necessary water to give the same slump varies between 33 and 37 gal per bag of cement," to "the necessary water to give the same slump varies between 33 and 37 gal per cu yd of concrete."

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[81] Cons. Engr. (Elwyn E. Seelye), New York, N. Y.

[81a] Received by the Secretary March 28, 1941.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EARTHQUAKE STRESSES IN THE SAN FRANCISCO-OAKLAND BAY BRIDGE

Discussion

By Leon S. Moisseiff, M. Am. Soc. C. E.

Leon S. Moisseiff,[8] M. Am. Soc. C. E. (by letter).[8a]—The scientific designing of structures is based on the analytical and experimental knowledge of the behavior of a given construction built of dependable materials of known strength characteristics and attached to a foundation of ascertained bearing resistance when subjected to the action of well-defined forces. Analyzing the foregoing statement one finds that it implies a number of major assumptions in knowledge. When dealing with the resistance of structures to earthquakes, the assumption made by the designer of the magnitude and kind of forces caused by earthquakes which may be encountered in the locality, and reasonably must be provided for in the structure, is of deciding importance. Engineers generally realize that the data on earthquakes are as yet meager and the knowledge of their effects is still vague. There are indications, however, that more extensive and well-planned observations which are being furthered at the present time, as well as deepened study, will tend to enlarge and precise the scope of information and furnish more assurance of the correctness of the assumptions made.

In the meantime, engineers who build structures in earthquake regions (and almost all regions are more or less vulnerable) are compelled to make the most of the available data on earthquakes. By making further simplifying assumptions as to the kind of action of the quake forces, their maximum effect is transformed mathematically into an equivalent horizontal force, the intensity of which is a fraction of the acceleration of gravity. For the location of the San Francisco-Oakland Bay Bridge this force was assumed to be 10% of gravity. In structural terms this means that a rigid body, which does not deform elastically, would be subjected to a horizontal force of 10% of its weight acting at the center of gravity.

Note.—This paper by Norman C. Raab, M. Am. Soc. C. E., and Howard C. Wood, Assoc. M. Am. Soc. C. E., was published in October, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Maurice A. Biot, Esq.; and March, 1941, by Franklin P. Ulrich, M. Am. Soc. C. E.

[8] Cons. Engr., New York, N. Y.; and Member of Board of Cons. Engrs., San Francisco-Oakland Bay Bridge.

[8a] Received by the Secretary March 31, 1941.

Attention is called to the fact that, in the present state of knowledge, the effect produced on the structure by an earthquake cannot as yet be considered as a "well-defined" force. Designers are compelled to content themselves with an assumed statical force. Having made this assumption, it becomes prudent to be somewhat lavish as to the intensity of the force, the more so when one finds that the provision for it in the structure causes relatively small expense.

What appears within the present knowledge of earthquake effects to be an ample intensity has been adopted in the design for the San Francisco-Oakland Bay Bridge. The pressure and stress determination procedure on the piers is very severe, as stated by the authors. The writer wishes to call attention to the extreme severity involved in the use of the dam analogy for piers surrounded by water. For the large piers of the San Francisco-Oakland Bay Bridge this procedure could well be afforded but this may not be the case with piers of other bridges. Engineers should not feel constrained to apply so severe a test. The writer is of the opinion that because water is present on both sides of the pier the acting forces need not be doubled.

The authors have done a service to the profession by presenting an expanded review of the treatment of earthquake effects in the San Francisco-Oakland Bay Bridge.

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DISCUSSIONS

RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

Discussion

BY MESSRS. WALDO E. SMITH, AND ROBERT L. LOWRY, JR.

WALDO E. SMITH,[33] M. AM. SOC. C. E. (by letter).[33a]—A welcome addition to the literature relating to the frequency of rainfall is found in this paper, which provides a statistical analysis of the frequently accepted and widely used station-year approach. It gives an index to the probable reliability of rainfall frequencies determined by that method and provides further an outline of a new method utilizing the basic statistical record. Mrs. Clarke-Hafstad is to be commended for her splendid presentation.

As the paper has its background in the mathematics of statistics, the major part of which the author has intentionally omitted, there are questions which arise that the reader is unable to answer. One of these is in connection with the statement under "Factors Affecting the Accuracy of Frequency Determinations: The Length of Independent Record" that "* * * there are 99.7 chances out of 100 that the mean computed for these 500 years * * * is not more than * * * 30% away from the true average * * *; and there are 68.3 chances out of 100 that this mean will not exceed or fall short of the true mean by more than * * * 10%." The writer believes that a little more explanation on the normal curve leading to the 99.7 and the 68.3 would not be out of place.

It appears that this method should have application to flood frequencies based on the historical record at a given gaging station. Inasmuch as this is a matter of treatment, station by station, to a large extent, data are even more meager, in a way, than for the determination of rainfall frequencies. With 100 years of historical record, it has been customary to call the largest flood within this period the 100-year flood. It is indeed quite disconcerting to find that this is correct to within the limits of $\pm \infty$, or (see heading mentioned previously) that "nothing actually is known concerning the frequency * * * from a single observation * * *." This approach does not take cognizance

NOTE.—This paper by Katharine Clarke-Hafstad was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by Paul V. Hodges, M. Am. Soc. C. E.; February, 1941, by Messrs. C. S. Jarvis, and Howard W. Brod; March 1941, by Messrs. Merrill Bernard, and Charles F. Ruff; and April, 1941, by Eugene L. Grant, M. Am. Soc. C. E.

[33] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[33a] Received by the Secretary March 28, 1941.

of the fact that this is not truly a single observation, but bears a certain relationship to others of lesser magnitude. Similarly, the second highest flood in such a record, usually given with some confidence as the 50-year flood (although doubt is acknowledged on the frequency of the greatest flood), has a standard error of frequency of $\pm$ 100% (this cannot be $-$ 100%). Inasmuch as the hydraulic engineer must, perforce, deal with the historic record and must base his designs on these elusive frequencies of great floods, this conclusion is likewise disturbing. It would seem more reasonable for this purpose to use the equation first given—namely, the standard error equals $\frac{1}{\sqrt{n}}$. This is probably nearer the truth. It acknowledges the possibility of a 100% error in the first value and about 70% in the second.

The symbol N_a is defined as the average number of stations affected by a storm of given intensity, whereas N_d equals the number of stations on the average receiving a given amount in each storm day. Assuming that a day is the period for the N_a-values, the definitions would appear to the writer to be identical. The author indicates that there are very limited conditions only for which this would be true, which the writer has been unable to follow. The wide disparity between the findings of the two values indicates a wide difference in concept that should be more fully explained. The fact that either N_a or N_d is used in evaluating the equivalent of a station-year record to a record of the same length, for a single station, would imply that the values should not be greatly divergent.

An interpretation of N_a or N_d, with regard to regions and seasons for a given region, is that they may be, to a limited extent, indexes of the rainfall

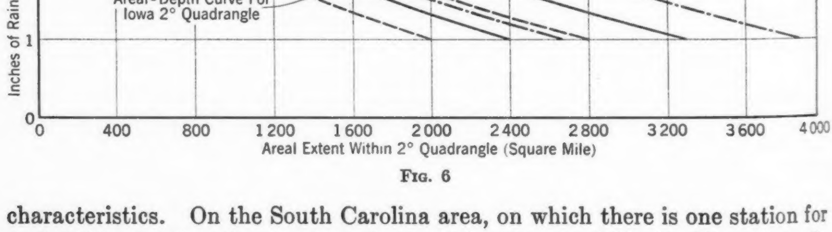

FIG. 6

characteristics. On the South Carolina area, on which there is one station for each 1,055 sq miles, the average extent of 1-in. storms within the 2° quadrangle is 3.10 $\pm$ 0.57 stations or 3,270 $\pm$ 600 sq miles. In Iowa there is one station for each 677 sq miles and the average 1-in. storm within the quadrangle covers 2,200 sq miles $\pm$ 390 sq miles. Determining these areas for the other daily rainfall rates given, and plotting them, define the curves shown in Fig. 6. Inasmuch as heavy rains are of more general extent in South Carolina than in

Iowa, one may conclude, at least tentatively, that they are affected less often by the purely convective influence. The plotting of the spread of the standard error gives an idea as to reasonableness of this value, determined on a statistical basis. It would appear in Fig. 6 that the standard error as found for 4 in. for South Carolina is definitely out of line with the others found, and one may feel some justification in applying an adjustment to the standard errors determined by the statistical method.

The author makes reference to the Republican River flood of May, 1935 (see heading "Factors Affecting the Accuracy of Frequency Determinations"), and states "that no gage recorded the rain storm which produced" the flood. An inspection of the published data[34] for this flood shows that several rain gages measured the rainfall, although it is true that no one of them secured the maximum catch indicated by certain unofficial measurements.

ROBERT L. LOWRY, JR.,[35] M. AM. SOC. C. E. (by letter).[35a]—It is fairly easy to prepare a diagram showing the "standard error" in relation to the number of observations for various frequencies. This has been done in Fig. 7, in which

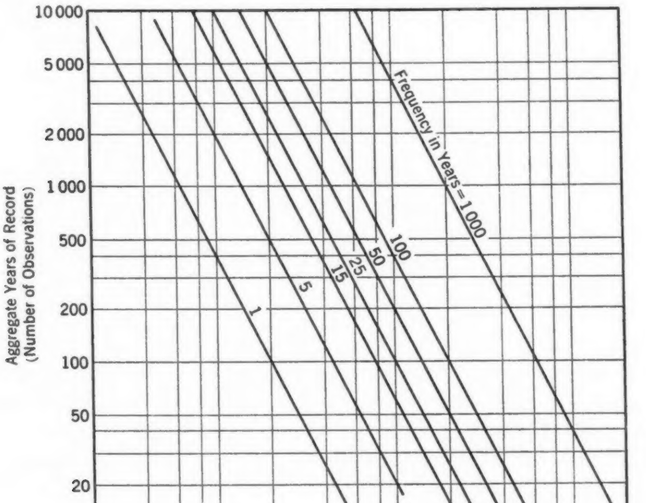

FIG. 7.—RELATION BETWEEN NUMBER OF OBSERVATIONS (IN THIS CASE, YEARS OF RECORD) AND THE STANDARD ERROR FOR GIVEN FREQUENCIES

the equations for the standard error plat as straight lines on double logarithmic paper. From such a chart the probable errors in frequency for any given conditions can be determined without computation. The chart shows at a glance the magnitude of the errors when only short records are considered.

[34] "Flood on Republican and Kansas Rivers, May and June, 1935," *Water Supply Paper No. 796-B*, U. S. Geological Survey, pp. 26–27.

[35] Hydr. Engr., National Resources Planning Board, Pecos River Joint Investigation, Roswell, N. Mex.

[35a] Received by the Secretary March 31, 1941.

However, other than the error as indicated, there is a more serious error in what seems to be the general practice in using the Miami Conservancy District Charts. Clearly, the author overlooked the proper interpretation of these charts. Under the discussion of "Standard Errors for Frequencies of Pluvial Indexes of the Miami Study" the following statement is made: "For each 2° quadrangle these charts give the maximum rainfall (one day to six days) to be expected once in 15 years, 25 years, 50 years, and 100 years." If the study referred to is examined it will be found that no such claim is made. In each case the pluvial index as indicated is that amount which may be expected to occur or to be exceeded within the quadrangle on an average of once in the frequency cited. Thus, the pluvial index may be far smaller than the probable precipitation to be expected within the frequency. At best it expresses only approximately the lower limit as to the precipitation to be expected. Certainly, it does not indicate the maximum.

Taking an example from the charts, the data for Quadrangle I–14 for a 3-day storm were plotted as shown in Fig. 8. The frequency curve defined by these

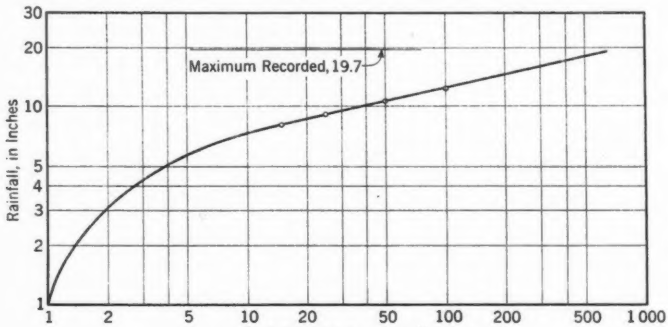

FIG 8.—DATA FROM MIAMI CHARTS FOR QUADRANGLE I–14 FOR A 3-DAY STORM

data has been extended in both directions purely for illustration. The interpretation of this curve is that the precipitation indicated for any frequency is that which may be expected to occur or be exceeded. By how much it may be exceeded is not known. That is where the trouble lies. Presumably, it could be anything up to the probable maximum that may be expected; but this probable maximum is also unknown. It is known, however, that for a 3-day storm in that quadrangle as much as 19.7 in. of precipitation has been recorded. Obviously, the maximum is some greater value.

From the charts it is not possible to determine what the maximum rainfall for any frequency may be. Yet it is the maximum that is required in hydraulic studies rather than the value indicated by the charts. Therefore, when the maximum is unknown, it seems that the probable error in the lower limit is of rather small consequence.

If more attention is given to the interpretation of the frequency curves and if judgment is applied in using them, frequency curves still will remain the best available guide as to the probable rainfall in the future. With some of their limitations pointed out, possibly better results may be expected in the future.

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DISCUSSIONS

FORT PECK SLIDE

Discussion

By Messrs. William Gerig, Alfred J. Ryan, and Glennon Gilboy

William Gerig,[13] M. Am. Soc. C. E. (by letter).[13a]—The excellent paper by Mr. Middlebrooks has covered the subject of the Fort Peck slide very well. In an effort to show the class of material placed in the dam as well as in the foundation, the author shows that much work in securing borings and other data was done in a thorough manner.

The writer has never experienced a case in which a "flow failure" has occurred in the dredge material. One who is familiar with dredges should know that sand handled by a dredge has practically all the clays and other sliding material removed, and placed on the core or washed off the dam, so that the particles of sand are compelled to slide on themselves, and, therefore, must have a high friction content. Dredged sand usually is a very closely packed material. As previously stated, the writer has never encountered a "flow failure" in dredged sand.

The material moved so far at Fort Peck, not because of a reduction in the strength of the material in the dam, but because of a reduction in strength in the foundation shale. The failure in the hydraulic fill, such as in a dam, is generally caused by an improper foundation, and sometimes by inferior material that is placed in the dam at improper slopes. The writer has never experienced such material as "partial liquefaction." When he signed the Report of the Board of Consultants,[14] he had in mind that the foundation material contained some clay, which might have been in a "state of liquefaction." In fact, when the word was first used in the Report, he did not know what it meant; but after the word was defined as being similar to what is usually called "quicksand," he thought the clays might flow.

In the Zanesville (Ohio) District (U. S. Engineer Office), for example, a part of a river was unwatered, and it was found that the material in the bed

Note.—This paper by T. A. Middlebrooks, Assoc. M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jacob Feld, M. Am. Soc. C. E.; and April, 1941, by Joel D. Justin, M. Am. Soc. C. E.

[13] Cons. Engr., Arkadelphia, Ark.

[13a] Received by the Secretary March 20, 1941.

[14] Report on the slide on a part of the upstream face of the Ft. Peck Dam, War Dept., Corps of Engrs., U. S. Army, 1939.

consisted of pebbles, small boulders, and sand, and that this was in a state similar to "quicksand." One cannot imagine that these boulders, in this condition, would liquefy in the sense that Webster defines it. The writer, therefore, fully indorses Mr. Middlebrooks' conception that the material in any part of the dam was not liquefied, and that the slide at Fort Peck Dam occurred in the foundation.

The conditions were quite similar (as far as could be seen) on both abutments, except that the downstream slope was 1 on 10, whereas that of the upstream was designed for 1 on 4.5 at the east abutment. The design was similar at the west abutment. There was little evidence of disintegrated shale at this abutment, however, because evidently there was a stronger current during high water, and this current had carried off the disintegrated shale. The west abutment, furthermore, was composed of a larger percentage of more or less firm glacial till.

Mr. Middlebrooks also mentions the 1-on-3 construction slopes in the closure of the dam where the river was used as an outlet for surplus water. This slope was not found stable in the test dams and was used only in the river section proper, so that when the closure was made the quantity of material required for the closure would be less than on a flatter section. When it was definitely decided that this slope was too steep, it was changed to about 1 on 4.5 and gave no more trouble.

If the failure on the east end of the dam was due to anything except a poor foundation (and probably the increased slope on the upstream side when the location of the dam was changed), it would have been evident on the downstream side. The bentonite seams were found in the tunnels, and they were not discharging a large quantity of water; in fact, there was no trouble with water in these tunnels. If the water in the bentonite seams were a contributing cause of the slide, it would have been shown in these seams, where they extended farther than the downstream side of the dam.

The author submits a statement showing the large number of borings and other data that were obtained prior to the construction of the dam and also the large total number that were obtained. Many of these data were duplicated after the slide occurred and were obtained chiefly as a check. The greatest possible care was taken to obtain the data correctly, and tests were carefully made and studied for this purpose. Often the data obtained by means of tests in a laboratory are not satisfactory. To illustrate: Obtaining the friction through various sizes of smooth pipe (approximately 8 in., 12 in., 20 in., and 30 in. in diameter), the frictional resistance per hundred feet was found to be different for the different sizes of pipe, although the velocities were kept the same in all sizes. This statement is inserted to show that entire dependence cannot always be given to the results of laboratory tests. This is true in many of the tests made in soil mechanics, and for that reason and others the writer views with doubt some of the results in soil mechanics obtained in this manner. Earth dams were built many years ago and some are still standing, although the theory of soil mechanics was not known to those builders.

ALFRED J. RYAN,[15] JUN. AM. SOC. C. E. (by letter).[15a]—The only true test of any method of analysis of earth embankments is to apply that method to the failure of an actual structure. Such an application is dependent upon knowledge of all of the facts pertaining to the failure. Mr. Middlebrooks' frank and complete description of the failure, and of the apparent reasons for the failure, at Fort Peck Dam is a valuable contribution to this phase of engineering.

The conclusion that the failure was the result of a weakened foundation seems sound beyond any reasonable doubt. If it is true that the static slide method of analysis, properly applied, would not indicate this failure, it is a matter of grave concern to those who have analyzed existing structures by that method. Any method of analysis, when applied to an embankment composed of more than one material, has some doubtful features. The static slide method is no exception, but it is no less applicable to the plastic flow condition than to any other condition.

The essential difference between the "elastic state" and the "plastic state" of soil is the existence of excessive hydrostatic pressure of the fluid contained in the voids of the latter. The fluid consists of water or air, or a mixture of the two. When a load is applied to a soil that has not been sufficiently consolidated, the soil particles attempt to adjust themselves to this load. This requires a reduction in the volume of voids. In a soil that resists drainage, the fluid in the voids cannot escape quickly enough to allow this reduction and part of the load must be carried by hydrostatic pressure of the fluid. It is commonly called "pore pressure" and cannot produce friction. As the fluid escapes, the volume of voids and the intensity of the pore pressure are reduced. The rate of reduction is a function of the permeability of the soil and of the distance to a drainage face. That part of the load not carried by the fluid (that is, the total load minus the pore pressure) is carried by the grain-to-grain contact of the soil particles and does produce friction. Therefore, the static slide method is adapted to a plastic condition by making the proper corrections for pore pressure. Methods of evaluating this pressure have been developed by Karl Terzaghi,[16] M. Am. Soc. C. E., and by J. H. A. Brahtz.[17]

The basic concept of the static slide method is that of an instantaneous failure along the entire sliding surface. Observed failures of both models and actual embankments have proved that this is not the case. Instead, the failure is progressive and the failure surface indicated by the analysis represents the approximate limit of this progression. A careful check as to the effect of this inconsistency reveals that the summation of the sliding forces and of the resistance can be represented by the basic concept as long as stability is maintained. Consequently, the fact that the conditions of an earth embankment are such that any failure will obviously be progressive does not exclude this method of analysis.

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[15] (Crocker & Ryan), Denver, Colo.

[15a] Received by the Secretary March 31, 1941.

[16] "Theorie der Setzung von Tonschichten eine Einführung in die Analytische Tonmechanik," von Karl Terzaghi und O. K. Fröhlich, Leipzig, Deuticke, 1936.

[17] "Notes on Soil Mechanics," *Technical Memorandum No. 592*, Bureau of Reclamation, U. S. Dept. of the Interior.

The writer has made a static slide analysis of the section with pore pressures computed by the method developed by Professor Terzaghi. The analysis was made at Station 15+00, as shown in Fig. 8, and was based upon information given by the author and in the report of the slide.[18] The consolidation characteristics of the soils were taken from the results of tests of representative samples of the various materials as shown in the report. The sample of foundation material was from the Merriman Drift and was classified as undisturbed weathered shale. The core sample was from Hole C–7. The shell material was assumed to be consolidated to the imposed load and to have no pore pressure. Due to lack of sufficient information, a check of the computed pore pressures by Mr. Brahtz' method was impossible. However, approximate agreement of these pressures with those shown in Fig. 1 was considered an adequate check. The computations indicated pore pressures in the weathered shale of sufficient intensity to make the shale, as well as the bentonite, susceptible to failure.

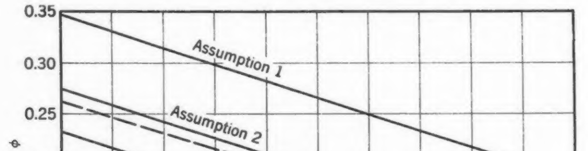

FIG. 15.—STRENGTH REQUIRED FOR STABILITY

Computations of the pore pressures in the core were complicated by the fact that the location and extent of the sand lenses were not known. Analyses were made for three assumptions regarding the effect of these lenses: (1) Neglecting the effect of the lenses; (2) assuming lenses such as to provide a 50% decrease in the distance between drainage faces; and (3) assuming lenses such that the pore pressures were completely dissipated at the time of the failure. The first and third assumptions were obviously used to establish limits and the second to indicate the intermediate trend. Comparison of computed consolidations along the center line of the core with those given in Fig. 5 indicate that the true condition lies somewhere between the first and second assumptions. Results of an analysis based upon the second assumption would be liberal instead of conservative.

[18] Report on the Slide of a Portion of the Upstream Face of the Fort Peck Dam, Corps of Engrs., U. S. Army.

Results of the analyses are shown in Fig. 15. The requirements for stability are slightly higher for the second assumption than those found by the author by the elastic theory method. In the writer's experience, this has usually been the case when comparing the results of the two methods. The author states that a static slide analysis will not indicate failure if a value of $\tan \phi = 0.6$ is used for the core material. But this analysis does indicate failure. With the exception of the pore pressures, the analysis was based entirely upon information given by the author or in the report, including the value of $\tan \phi = 0.6$ for the core material. Only two differences can exist: (1) The generalization of using the values of one representative sample for each material; and (2) the pore pressures. All embankment analyses are based upon a degree of generalization, so any difference from that source should be small enough to be negligible. Apparently then, the adaptation of the static slide method of analysis to this type of dam is dependent upon the correct application of the pore pressures, and these pressures were the only conditions not fully covered by the author.

Therefore, it is felt that if more complete information along this line were included in the concluding discussion, it would be of definite value to all of those interested in the design of earth embankments.

Glennon Gilboy,[19] Assoc. M. Am. Soc. C. E. (by letter).[19a]—The principal results of an unusually complicated and diversified investigation have been condensed and put in readable form by Mr. Middlebrooks. The writer wishes to add a few remarks of general interest, and to express certain opinions that are somewhat at variance with those of the author.

In terms of the size of an individual man, or even of the size of the massive construction equipment used, the Fort Peck Slide was a gigantic affair; but in terms of the size of the project it was a relatively minor localized disturbance. Specifically, the volume of earth moved was only 5% of the total volume in place at the time, and the length affected was about 8% of the total length. The sense of proportion associated with these percentages is, in the writer's opinion, an important element in any study of the problem.

An excellent picture of the localized nature of the disturbance is given in Fig. 2. The distortion of the upstream shell at the right (westerly) edge of the break shows clearly that the dam at this point had sufficient reserve strength to offer effective resistance against further propagation of the movement. The cross section here was no stronger than that in the slide area. Hence arises the logical inference that the dam itself was stable under the existing conditions, and that a weakness of the foundation, strictly localized in the vicinity of the right abutment, must have been responsible for the movement.

Fig. 1 illustrates the manner in which the foundation near the right abutment differed materially from that in the major part of the valley. The shale surface rises to a flat bench at a relatively shallow elevation, extending approximately from Station 13 to Station 19. A considerable depth of weathering is

[19] Cons. Engr., Lincoln, Mass.

[19a] Received by the Secretary April 15, 1941.

shown and the depth of penetration of the sheet piling indicates low resistance in the surface shale. Furthermore, the shale is interspersed with seams of bentonite, which tend to decrease its shearing resistance, especially in a horizontal direction.

Mr. Middlebrooks also mentions the intake channel, 800 ft upstream from the toe, and cut into the shale to El. 2,030. A significant detail in this connection is that the channel was cut approximately parallel to the dam from the intake structure, Station 11, out to Station 19, from which it veered sharply upstream.

Thus it is clear that a substantial length of the dam near the right abutment rested on a shelf of weathered shale and bentonite which, on account of its shallow depth, was subjected to the full intensity of horizontal shear induced by the embankment, and which was horizontally unsupported as a result of the channel excavation.

It is the writer's belief that the shearing strength of the material composing this shelf, though undoubtedly increasing somewhat under the weight of the dam, did not increase fast enough to keep pace with the increasing horizontal shear; that the margin of safety was thus decreased as the height of the dam increased; that the balance point was reached when there was still about 25 ft to go; and that movement was initiated by an essentially horizontal shear failure of the shelf material in the direction of its unsupported face.

The observed facts were consistent with this hypothesis. Among the more significant are:

(1) The failure was not sudden, but gradual. Distortions visible to the unaided eye were noted more than three hours before the slide took place. How much strain had developed previously it is impossible to say; but a considerable amount could have gone undetected due to the very nature of the operation.

(2) The distortions were maximal between Station 15 and Station 17, the precise center of the critical zone previously described.

(3) The first indication of the rapid final phase of the slide was a settling of the core pool in this zone.

(4) Initially, the upstream shell moved out practically all at once, like a huge gate hinged at the abutment.

(5) Subsequent investigations showed masses of weathered shale to be partially or wholly absent from their positions as determined by earlier borings.

So much for the initial phase. It should be noted here that the foregoing concept is the writer's personal explanation, and that, although the Board of Consultants agreed unanimously on the weathered shale zone as the source of the trouble, the detailed concepts of the individual members may differ materially from that presented here.

As to the progress of the slide after the initial failure, no unanimity of opinion was obtainable, and the Board was obliged to leave the question undecided in its final report. Since the writer's opinion on this point does not coincide with that expressed by the author, an explanation of the reasoning behind it seems indicated.

The upstream movement of material was truly remarkable. The intake channel, 800 ft upstream from the toe and 500 ft wide, was completely filled, to depths as great as 40 ft. Fanning out of material toward the center of the valley was on a comparable scale. Intact masses of the upstream shell moved more than a thousand feet from their original positions. The author's explanation of this extensive movement is that "liquefaction" took place in the weathered shale and bentonite zone, due to trapped water. This is equivalent to stating that the shearing strength of this zone reached a maximum at the instant of initial failure, and decreased to a considerably lower value after the peak point was passed. The writer does not dispute the probability of such reduction in shearing strength, but submits that by itself it is not sufficient to account for the observed behavior of the slide.

Suppose the dam had been constructed by rolled-fill methods. Horizontal base shears would have existed, of course. Suppose further that the dimensions of the rolled-fill structure and its rate of construction had been such as to bring about a balance of shearing stress and shearing strength in the weathered foundation zone when the dam was within 25 ft of the top. Shear failure in the foundation would have taken place. Reduction of shearing strength beyond the peak point would have exaggerated the movement. Considerable damage would have resulted, and expensive repairs would have been required. However it is inconceivable, to the writer's mind at least, that the rolled-fill material would have moved to anything like the extent observed in the actual slide. Outward movement of the toe, severe slumping in the main body of the dam, cracking, fissuring, faulting, all these things the writer can visualize; but nothing like an upstream movement of more than a thousand feet.

If this visualization is sound, it follows that the progress of the actual slide was intimately associated with the peculiarities of hydraulic-fill construction. Among these are: Complete saturation of the central part of the dam, with an excess of water on top; segregation of progressively finer materials from the outer slopes toward the core; and no artificial compaction. When construction is progressing normally, these items are merely part of the program; but if a disturbance occurs, they can unleash latent forces within the body of the dam which will tend to exaggerate the results of the disturbance.

Consider the effect of the core pool alone. If a shear failure of the foundation occurs, removal of support from the core will cause it to slump. A rolled-fill core would slump under similar circumstances; but when a hydraulic core slumps, the water of the core pool piles up at the low spot and produces an additional push on top of the core, thereby exaggerating the movement of the slide. This action will continue until the shell is fissured, whereupon the accumulated pool water will rush out through the fissures, carrying masses o core and shell material with it.

Segregation of fine, saturated, sandy material in the vicinity of the core—in the section of the dam often termed the "transition zone"—is another potential source of hazard. The coarser parts of the shell are inherently stable, or can be made so by artificial compaction; but the transition zone, half in and half out of water, is difficult to treat. A violent disturbance will tend to produce a temporary "quick" or "liquefied" condition in this zone, with a consequent increase of internal pressure and a further tendency to exaggerate the effect of the disturbance.

The writer submits that both the foregoing influences are present in any hydraulic-fill dam; that they were present in the Fort Peck Dam; that the initial disturbance due to foundation failure was violent enough to bring them fully into play; and that they were primarily responsible for the great distance covered by the slide after the initial failure.

The author dismisses the possibility of liquefaction of the shell on the ground that subsequent tests (Fig. 6) showed that the shell was denser than the critical value obtained for most of the samples tested. The writer agrees that most of the shell, certainly the outer parts, behaved very well; but this is no proof that liquefaction of the transition zone did not take place. On the contrary, at the time of the writer's first inspection, ten days after the slide, much of the surface material in the slide zone, especially the fine sand, was still in a "quick" condition, and it is difficult to imagine that it was not so during the occurrence of the slide. The amount of material which liquefied might have been small in comparison with the volume of the slide, yet might have played a very important rôle in keeping the more stable masses on the move.

As far as the critical density tests are concerned, it will be noted that the density of the fill coincides closely with the "minimum critical density" curve of Fig. 6. This means that the margin of safety was not very great. Although it is true, as the author states, that sands looser than critical may exhibit some residual shearing strength when disturbed, the writer believes that the nature and violence of the disturbance are of great importance, and that saturated sands somewhat denser than critical may, if sufficiently disturbed, pass through a temporary condition of liquefaction before attaining a new condition of equilibrium. Therefore, the writer contends that the critical density tests do not prove the absence of liquefaction; and that the balance of other evidence is strongly in favor of the view that some liquefaction took place, especially in the transition zone, and assisted in the movement of the slide.

In view of these considerations, the writer is now of the opinion that his method for computing shell stability does not contain as great a hidden factor of safety as might at first glance be expected. To provide absolute safety under all conditions, the parts of the shells which are inherently stable when deposited, or can be made so by artificial compaction, should be strong enough to withstand possible liquid pressure from the transition zone which might be produced by subsequent disturbances. While this may seem an unnecessarily severe criterion, it must be remembered that as improvements in design methods permit increasing economies in cross section, the influences of previously negligible factors become increasingly important.

In conclusion, the writer wishes to endorse the views of the author concerning the importance of thorough quantitative investigations of the properties of weak rocks such as those encountered at Fort Peck. These materials fall into a sort of twilight zone, where soil mechanics ends and geology begins, and the combined efforts of workers in both subjects should go far toward reducing the uncertainties now existing in this field.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EXPANSION OF CONCRETE THROUGH REACTION BETWEEN CEMENT AND AGGREGATE

Discussion

BY MESSRS. W. C. HANNA, J. C. WITT, AND R. F. BLANKS

W. C. HANNA,[15] ESQ. (by letter).[15a]—That very reactive aggregates exist in certain districts of California has been known for many years. The occasional defective concrete constructions made with these aggregates have probably contained cements of all brands used within the state, and this includes cements manufactured in foreign countries.

As early as 1934, the company with which the writer is connected investigated the case given by Mr. Stanton in Fig. 1(*d*) (a building completed about 1931), recognized the abnormal condition, and decided to determine the cause. The cement used in the construction was of most excellent quality, sound in every respect, and with both the manufacturer's tests and the independent inspection of a well-known commercial laboratory to prove the quality. A study was started at once and the company assigned Ira C. Bechtold (later, Portland Cement Association Fellowship, National Bureau of Standards) to devote considerable time to the problem.

Inspection of concrete cores from this structure revealed the gel formation which evidently had caused the cracks. From the study of the concrete it was possible to trace the bad aggregate to its source and to collect specimens of it for laboratory study. Photographs of the concrete showing the gel formation, together with Mr. Bechtold's findings, were exhibited to a number of organizations and individuals who were interested in the matter. This discussion is based on the resultant report.

Examination of the concrete disclosed that it contained many particles that were evidently foreign to the remainder of the aggregate material. These particles ranged in color from a dark brown to a light tan or buff. They were quite hard, but brittle, when dry and intact. They were sometimes cracked

NOTE.—This paper by Thomas E. Stanton, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by R. W. Carlson, Assoc. M. Am. Soc. C. E.; March, 1941, by Bailey Tremper, Esq.; and April, 1941, by Messrs. Hubert Woods, and N. T. Stadtfeld.

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[15a] Received by the Secretary March 18, 1941.

and frequently disintegrated and contracted in the center. Practically all such particles showed a hard, shiny shell, and in some cases small spheres were noted near them which appeared to be hard and white but brittle when dry. These particles varied in size from very small grains to pieces about $\frac{1}{4}$ in. by $\frac{1}{4}$ in.

To this point all specimens were kept dry. Then they were immersed in water for a period of three days, after which they were removed, allowed to dry at room temperature for three days, and then again immersed in water. After twenty-four hours in water the foreign particles became very soft and began to disintegrate; and from many of them a definite formation of gel occurred, in some cases in a quantity almost equal to the volume of the original particle. The envelope around many particles became wet and soft but did not expand appreciably. The aforementioned spheres also softened. Some of the particles which showed marked disintegration before wetting did not form large amounts of gel. In one case a specimen from a core developed a large amount of gel from an area that became cracked as the piece remained wet; and finally a small piece of concrete was forced away by the gel.

Some cracks in concrete specimens from cores were found to pass entirely through the specimen. In several such cases it was found that gel was being squeezed out of the crack, sometimes in several spots along the crack. Where there was a crack it was always found that there was at least one particle developing gel within it.

An attempt was made to determine the chemical nature of the gelatinous material being formed from the particles under consideration. It was almost impossible to secure the pure gel material for test purposes because, in the process of formation, it always became contaminated with cement and small fragments of aggregate. Furthermore, it is to be expected that this type of gel would absorb calcium hydroxide and other material from the water around it. However, the following approximate analysis was obtained on a small portion of relatively clean gel scraped from the specimen: SiO_2, 41%; Al_2O_3, 11%; CaO, 33%; and H_2O, 17%.

Mr. Bechtold expressed an opinion that reaction of the calcium hydroxide freed from the cement by water is involved in the formation of this gel. The gel formation appears to be irreversible. Evidently, the gel progressively swells after it is first formed, but fortunately, after once being dried and contracted, it will not swell again when brought into contact with water. This point on the irreversibility of this reaction is important. It indicates that during the life history of a given piece of concrete construction there is a gradual slowing up of the reaction between calcium hydroxide and the reactive aggregate. In the early stages this reaction may be severe enough to cause cracking and "pop outs," but after a period of several years, depending on how reactive the aggregate happens to be, the disruptive reactions will have spent themselves.

Further research reported in 1936 indicated that volume changes and the extent of cracking evidenced in the ordinary laboratory specimens were not indicative of the intensive destructive forces that appeared to be responsible for the failures in question. Laboratory specimens will not indicate the same extent of change observed in the field. The expansive force of the gel can derange the particles of concrete so much that readjustment to the former position is impossible. When such readjustment is forced by shrinkage under drying conditions, the deranged particles may act as wedges to force the cracks open.

The shales vary tremendously in activity. They have not been found to be active to water alone to the extent of forming gel, although disintegration has been observed. In saturated limewater many of them produce large quantities of gel. In some cases the gel produced has been estimated to have a volume of 6 to 8 times that of the shale producing it. Other specimens of apparently the same material produce little gel and some show no sign of activity. Concentrations of even less than 10% of shale could produce stresses that would exceed the tensile strength of the strongest concrete.

At least a part of the bad aggregates in the aforementioned structure produced gel with limewater which, of course, is found in all normal concrete. There was a sufficient amount of such aggregate in the concrete to account for the damage. The gel formed in the concrete did not indicate that the alkalis of sodium and potassium were necessary for the reaction.

Mr. Stanton's bad aggregate is not the same as those detected in the aforementioned structure. It seems to resist the limewater but is readily attacked by a 10% solution of either K_2SO_4 or KOH. It was of interest to note that the reaction with these potassium compounds was more severe than with the corresponding sodium compounds.

The Stanton test, using sealed containers, is a valuable one for estimating the relative activity of certain California aggregates. For the present, the test should be classed as specific, because the writer has records of aggregates which have proved to be unsatisfactory in the field but which failed to show any excessive expansion when subjected to a Stanton test lasting several months. Mr. Stanton was admittedly dealing with a bad aggregate as revealed both in field work and by his unique laboratory test. All users of concrete should recognize his contribution in developing a test that is easily made and that certainly reveals doubtful quality of the aggregate he was testing.

Mr. Stanton has indicated that expansions he has encountered may be due in part to a reaction between $MgCO_3$ and $NaOH$. It is indicated that $Na_2CO_3 \cdot 10\,H_2O$ is a product of this reaction and that, due to the addition of ten molecules of water to the Na_2CO_3, an enormous increase in volume results. This in turn has a disruptive effect on the concrete. The writer is convinced that this possible reaction does not explain the expansion effects encountered by Mr. Stanton. From a practical standpoint, it is known that dolomitic limestone or magnesian limestone, when free of reactive siliceous material, is classed as one of the best of aggregates. Many widely distributed concrete structures, thirty or forty years old and containing dolomitic limestone as aggregate, are in excellent condition. It seems logical to conclude that the magnesian limestone part of this bad aggregate is not at fault. Some instability of the siliceous phase of this aggregate may be the cause of the failures reported by Mr. Stanton.

A close study of the paper will reveal a sincere attempt to establish the limitations of concrete made from portland cement, aggregate, and water. A clear conception of the function and limitation of each ingredient entering into concrete is essential.

Portland cement is very reactive chemically, and must be so to be useful. The compound composition of cement is complex, and the reactions taking place upon hydration are involved. The products of reaction are basic in character. The basic compounds may react within the cement composition itself or with some added material such as reactive aggregate. Aggregates that are reactive to portland cement compounds enter the field of puzzolanic materials. Some materials that may be definitely classed as poor and unsound aggregates, upon being interground with portland cement, will produce a sound and high-quality puzzolanic cement. In some respects there is a relation between the reaction products of portland cement and the reaction products from unsound aggregates. Both are active chemically and yield complex calcium silicates. Placing a relatively large particle of reactive aggregate within a concrete structure is always dangerous. The reactivity of the piece of aggregate may persist for an indefinite period, and there is always danger of failure of concrete, if the destructive forces set up are sufficient to overcome the inherent strength of the concrete structure. Construction engineers should attempt to approach the ideal in formulating concrete; that is, they should use a high quality portland cement plus a properly graded inert aggregate. This thought, of course, is not a new one; but it is worth repeating.

The record of failure as reported by Mr. Stanton is conclusive and indicates that constructive work toward improvement must be done by all concerned. The cement manufacturer should be required to produce the best quality of cement consistent with available raw materials. Great care should be taken as to the quality of aggregate, and the governing factors are relative inertness, high inherent strength, proper size distribution, and reasonable availability to the construction site. Further constructive work must be done in the field of physical and chemical testing of cements and aggregates. It is difficult to devise practical tests of this kind, but the work should be continued until satisfactory engineering tests are developed.

The field of better design for concrete structures should not be overlooked. There is still plenty of room for improvement in the design of concrete structures. Another study of a more fundamental nature involves a better knowledge of the chemistry of cement compounds. Considerable is known of the function of the recognized oxides in cement, such as silica, iron oxide, alumina, lime, magnesia, and sulfuric anhydride. Small amounts of other materials such as titanium dioxide, phosphorus compounds, and alkalis are present in all commercial cements. The effect of these incidental materials should be the subject of further study.

The chemical form of the sodium and potassium compounds present in cement is not known. There is evidence that potassium in cement often occurs as K_2SO_4. The possibility of complex calcium-sodium silicates should not be overlooked; nor should the alkalis in the aggregates be forgotten. The writer has made many Stanton tests in which 1% sodium or potassium hydroxide was deliberately added to the normal portland cement used in the test. In general, the addition of active alkali appeared to lower, or had little effect on, the degree of expansion.

The time has not come to establish a definite specification limit for alkali in portland cement. Considerable difficulty has been encountered in obtaining check results on alkali determinations. The writer has seen a great discrepancy in results on check samples submitted to representative laboratories. Considerable cooperative work is being done on standardizing the methods for estimating sodium and potassium oxides in portland cement. In the future, chemists probably will be able to estimate these compounds with reasonable certainty, but for the present they are inclined to allow considerable tolerance in any estimate of alkalis.

Out of the maze of discussion that has resulted from the commendable work of Mr. Stanton and others on alkali in portland cement, much good will result. Every one seems to be "alkali conscious," and the writer considers the current agitation as a "one track" proposition. All factors that can lead to unsound concrete should be investigated carefully. Cement manufacturers will do their utmost to cooperate with every one interested in the fabrication of better concrete structures.

J. C. WITT,[16] ESQ. (by letter).[16a]—In his very interesting paper Mr. Stanton presents some field and laboratory data on the expansion of concrete. On the basis of fact and assumption he points out relations that may exist between the two groups of phenomena. However, some statements are made with a "q.e.d.-ness," or finality, which the data do not justify. Reference to a few points will be sufficient to make this clear.

Under the heading "Earlier Failures" this statement appears:

> "Although the sulfates in sea water undoubtedly contributed to the serious deterioration of the sea walls, it was obvious that this deterioration was accelerated by the infiltration of sea water through cracks resulting from excessive expansion occasioned by other causes."

The deterioration of concrete exposed to sulfate solutions has been observed all over the world. It has not been restricted to cements or aggregates of any particular composition, nor to any combination of the two. The reason is not apparent why the deterioration of the sea walls mentioned in the paper is due primarily to the nature of the cement and aggregate present—the sea water simply being a contributing agent.

In the laboratory, specimens containing cements relatively high in alkali compounds and certain fine aggregates were subjected to storage conditions involving a wide range of moisture and temperature. After more than a year no excessive expansion was observed. Accidentally it was found that expansion could be obtained easily by placing specimens in sealed containers. This procedure does not represent the type of exposure to which most concrete is subjected.

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[16a] Received by the Secretary March 20, 1941.

The following statement is quoted from the author's discussion of Table 1:

> "It has not been determined if the reaction in the case of this impure limestone is due entirely to some form of silica or in part to the magnesium carbonate. If it is due in any part to the magnesium carbonate, the probable chemical reactions in this respect are * * *."

This is followed by a computation (Eq. 1) indicating that, when magnesium carbonate and sodium hydroxide react, the products are magnesium hydroxide and sodium carbonate (with ten molecules of water of crystallization), accompanied by an increase in volume of 239%. Apparently this is the basis for such statements in the "Conclusion" as: "Certain mineral constituents in concrete aggregates contribute to expansion of concrete and sometimes develop stresses of such magnitude as to cause failure," and "The chemical reaction producing excessive expansion apparently occurs only when the portland cement component contains an appreciable percentage of alkali in the form of sodium and potassium oxides."

Here are a few of the comments that could be made:

(*a*) The volume of the water of crystallization is not included in the "combined volume of reacting compounds." In conformity with the author's nomenclature, this volume is 180.16. It is equivalent to 276% of the combined volume of the listed reacting compounds and to 80% of the reaction products.

(*b*) There is a series of hydrous sodium carbonates. There is no basis for assuming that $Na_2CO_3 \cdot 10\, H_2O$ alone would crystallize from solution—the only manner in which any member of the series is formed. A crystal containing a given number of molecules of water does not develop directly into a crystal containing more water. Re-solution and recrystallization are necessary. The compound $Na_2CO_3 \cdot 10\, H_2O$ is unstable—losing water when in contact with air.

(*c*) If crystallization occurs when the cement paste is plastic, no cracking of a specimen would be expected. After the cement has hardened, very little water is available in the interior of a specimen stored in a sealed container. At the surface some water is available, but crystallization here would not cause cracks of any importance.

Following Eq. 1 this statement appears with reference to the volume increase computations: "To check the accuracy of this assumption, tests were made to determine the action of sodium hydroxide upon the various materials that might be present in the aggregate, * * *." Several substances were treated with a large excess of normal sodium hydroxide solution. The conditions differed greatly from the other tests described in the paper, and the results do not serve to check any of the assumptions.

R. F. BLANKS,[17] M. AM. SOC. C. E. (by letter).[17a]—The far-reaching discovery, reported by Mr. Stanton, that reactions between alkalis in cement and some mineral constituents of aggregates explain certain cases of concrete expansion, cracking, and disintegration is not surprising. Since about 1930,

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[17a] Received by the Secretary April 7, 1941.

there has been a growing suspicion that the minor compounds in cement (including alkalis) might have a greater influence on the quality of cement and the resulting concrete than was generally realized. The incredible part, in the light of developments since Mr. Stanton's discovery, is that it has taken engineers and investigators so long to arrive at this explanation.

The Constituent Materials in Concrete.—During recent years the quality of portland cement has been improved notably, and this has been reflected in specification requirements. Limitations on various compounds that have been found to be objectionable in portland cement have been imposed, and in some cases manufacturing methods and procedure have been limited by specification requirements. Free lime, magnesia, alumina, and, in special cases, iron, have been subjected to varying degrees of restriction. During the past few months (1941) the alkalis of sodium and potassium have likewise been limited in some special cases. However, far too little is yet known regarding the true constitution of portland cement, in what forms the various compounds really exist, or what reactions are actually involved in the hydration processes.

It cannot be said that even comparable progress has been made in the specification and use of aggregate materials. The popular conception that aggregates are merely inert filler materials and that most all rocks, with a few exceptions, will make good concrete still persists. However, as P. H. Bates has indicated,[18] all aggregate materials are subject to chemical reaction in concrete and the real problem is whether or not they are adversely reactive. Mr. Bates also pointed out that the compounds of calcium, as well as of sodium and potassium, should not be overlooked in searching for the solutions to the alkali-aggregate reaction problems. Present criteria by which the suitability of aggregates for use in concrete is judged are woefully inadequate, and basic information for intelligent guidance is almost entirely lacking.

In the light of the aforementioned conditions, it is again not surprising that, since Mr. Stanton's discovery, many cases of concrete disintegration similar to those that he described have been revealed. Concrete structures either known to be or strongly suspected of suffering from alkali-aggregate reactions are now under observation in other parts of California, and in Arizona, New Mexico, Washington, Idaho, Oregon, Nebraska, Canada, Iowa, Virginia, and Pennsylvania. The list of aggregates known to be reactive in the presence of alkalis includes materials having such descriptive terms as andesitic, rhyolitic, felsitic, granitic, slaty, schistose, impure limestones, and others as yet unidentified, in addition to the shale, chert, and siliceous limestone reported by Mr. Stanton.

Preventive Measures.—The predominating question in the minds of all who are concerned with the alkali-aggregate reaction problems, of course, is "How can future cases of such concrete disintegration be avoided?" From the cement manufacturer's viewpoint, alkali-reactive aggregates should be excluded from use. He substantiates his position by stating that most of the concrete in existence is rendering satisfactory service even though high-alkali cement was used in much of that concrete. Concrete investigators emphasize that there is, as yet, no certain method of determining what combination of cement

[18] *Proceedings*, Conferences on Problems Related to Alkalies in Cement and Their Effect on Aggregates and Concretes, Bureau of Reclamation, February, 1941, p. 20.

and aggregate might be expected to cause trouble under various service conditions. Furthermore, the occurrence of alkali-reactive aggregates is not a condition peculiar to any one locality but is apparently widespread.

The engineer and builder, in some cases, finds himself in a quandary. For most ordinary concrete work, particularly where aggregates with long service records in combination with a relatively high-alkali cement are available, the problems probably need to occasion little concern, except an uneasy realization that alkalis in cement are not desirable constituents. For large, important, and costly works there is, for the moment, only one available expedient—the imposition of the lowest practicable limit on the alkali content of the cement.

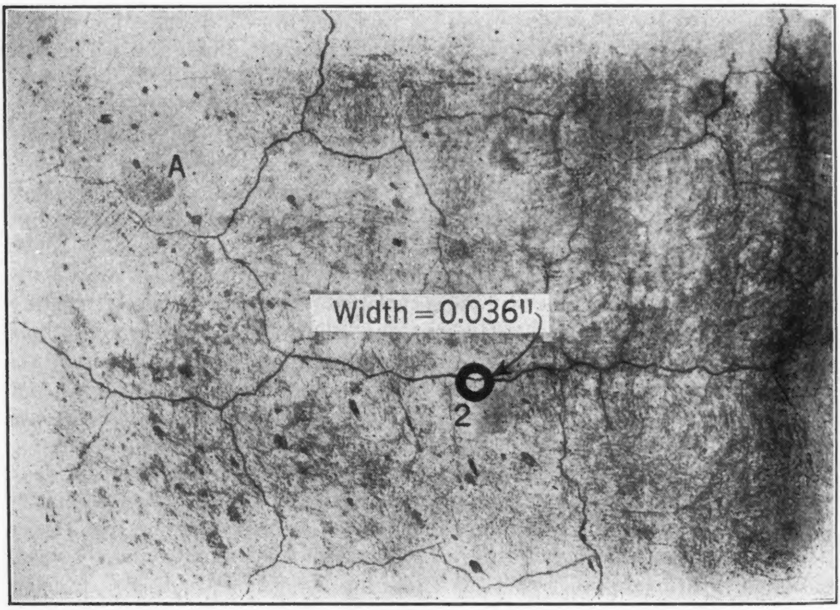

FIG. 10.—TYPICAL RANDOM-PATTERN CRACKING OF THE PARKER DAM SURFACE CONCRETE AS A RESULT OF INTERNAL EXPANSION

Whether this expedient affords adequate protection in all cases, whether some aggregates should be condemned for use in concrete, or whether there are other more effective corrective measures whereby the destructive chemical reactions can be neutralized yet remains to be determined.

Comparison of Mr. Stanton's Findings with Those of the Bureau of Reclamation at Parker Dam.—The Parker Dam case has been discussed in detail in the technical press.[19,20] It is sufficient to state here that Parker Dam was completed in 1938, and, after less than two years of service, random-pattern cracking (Fig. 10) and internal expansion of the concrete developed on an extensive

[19] "Concrete Deterioration at Parker Dam," by R. F. Blanks, *Engineering News-Record*, March 27, 1941, p. 46.

[20] "Cracking in Concrete Due to Expansive Reaction Between Aggregate and High-Alkali Cement As Evidenced at Parker Dam," by H. S. Meissner, Assoc. M. Am. Soc. C. E., *Journal*, A. C. I., April, 1941.

scale. As high-alkali cement was used in the construction of Parker Dam, the possibility of alkali-aggregate reactions, similar to those described by Mr. Stanton, was immediately investigated by means of his 1 : 3 mortar-bar expansion test at 70° F. The results obtained from such tests at six months (Fig. 11(*a*)) show about the same expansion for the Parker sand, Grand Coulee sand, and the local Denver sand from the Platte River (approximately 300 millionths) with a cement containing 1.13% total alkalis. In comparison with

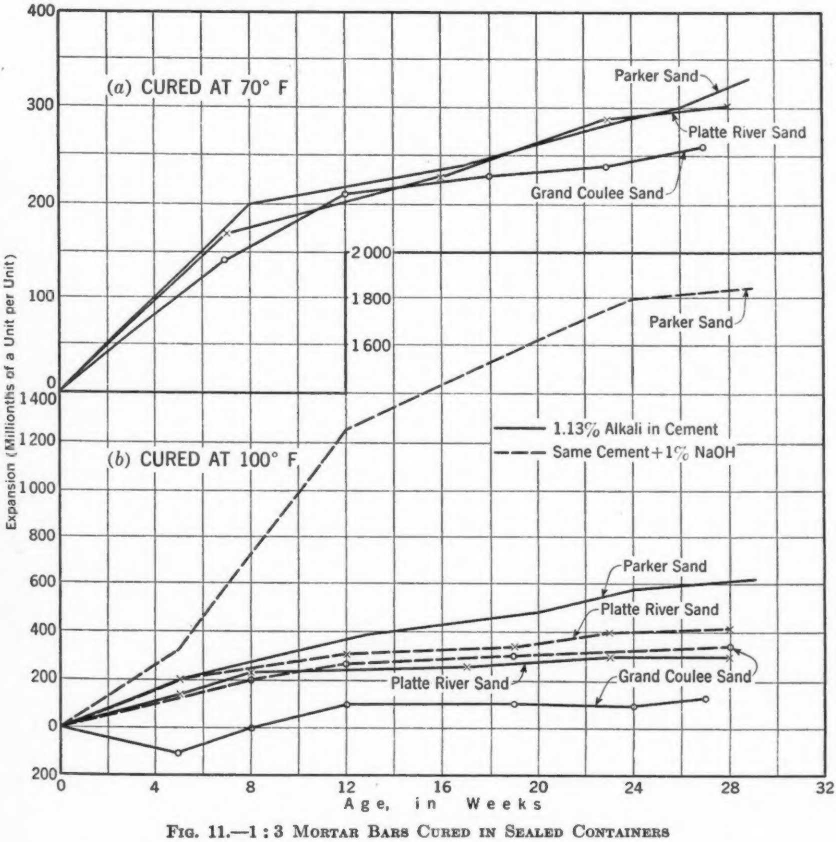

FIG. 11.—1 : 3 MORTAR BARS CURED IN SEALED CONTAINERS

Fig. 4, this might be interpreted as meaning that all the aforementioned sands are mildly reactive in combination with a high-alkali cement. On the other hand, the foregoing results might be interpreted as meaning that none of these sands are reactive, the recorded uniform expansions being due simply to increased moisture content of the mortar bars.

Conclusive evidence to explain the condition of the Parker Dam concrete was not obtained until cores were drilled. When the Parker concrete cores were stored in the presence of moisture, a gel-like ooze appeared on their surfaces and also was found in the small cavities and voids within the concrete mass (see Fig. 12). Upon drying, this gel became a hard whitish deposit similar to that described by Mr. Stanton. Chemical analysis of the gel revealed it to be essentially sodium silicate. Further investigation showed that the gel-like substance was apparently a reaction product between the alkalis in the cement and as yet unidentified mineral constituents carried by small percentages of andesitic and rhyolitic aggregate particles in the Parker gravel (see Fig. 13). Whereas it is inferred from Mr. Stanton's paper that only the sand in the California aggregates carried alkali-reactive materials, 6-in. cobbles have been found undergoing chemical alteration in the Parker concrete.

After the reactive aggregates in the Parker gravel were identified by means of the concrete-core studies, samples of the andesite rocks were hand picked and the material subjected to tests corresponding to those shown in Fig. 6(*b*) for siliceous limestone. The comparative responses of a 10% crushed Parker andesite and the 5% siliceous magnesian California limestone (as reported by Mr. Stanton) to the mortar-bar expansion test at 70° F are shown in Figs. 14(*b*) and 14(*a*), respectively.

In an effort to develop an accelerated test for detecting alkali-reactive aggregates, the mortar-bar test was conducted at 100° F with a cement containing 1.13% alkalis and also with the same cement to which had been added 1% of Na OH. The results from these tests are shown in Fig. 11(*b*). Again, the correct interpretation of these results is not very clear. However, the differences in the expansions obtained with and without the addition of Na OH are believed to be significant. A close comparison of the data shown in Fig. 11(*b*) with those represented in Fig. 11(*a*) also indicates some inconsistencies.

The most significant fact developed by Mr. Stanton's investigations, corroborated by those conducted by the Bureau of Reclamation, is that "bad" aggregates apparently make "good" concrete in combination with cements containing less than 0.60% total alkalis. The question immediately arises: "Is an aggregate that is reactive in the presence of a high-alkali cement 'bad' or is the high-alkali cement 'bad'?" This is an important question that has not as yet been answered satisfactorily.

For concrete work in those areas where alkali-reactive aggregates are known to occur, the California Highway Department has limited the alkali content of cement to 0.60%. The Bureau of Reclamation has imposed such a limit on cements being used in current large concrete works. Practically all of the natural sand and gravel aggregates available in the Western States contain varying percentages of andesitic, rhyolitic, and felsitic materials. Not all particles of andesite and rhyolite found in the Parker concrete show evidence of chemical alteration and, likewise, examples of felsite in concrete containing high-alkali cement have been observed to be chemically inactive. However, methods for predetermining when such materials may or may not be reactive in the presence of excessive quantities of alkalis in cement have not been discovered. It is reasonably certain that the reactive minerals in these aggregate materials are products of alteration, and the intensive petrographic and microscopic investigations of aggregates, cements, and concretes that are being

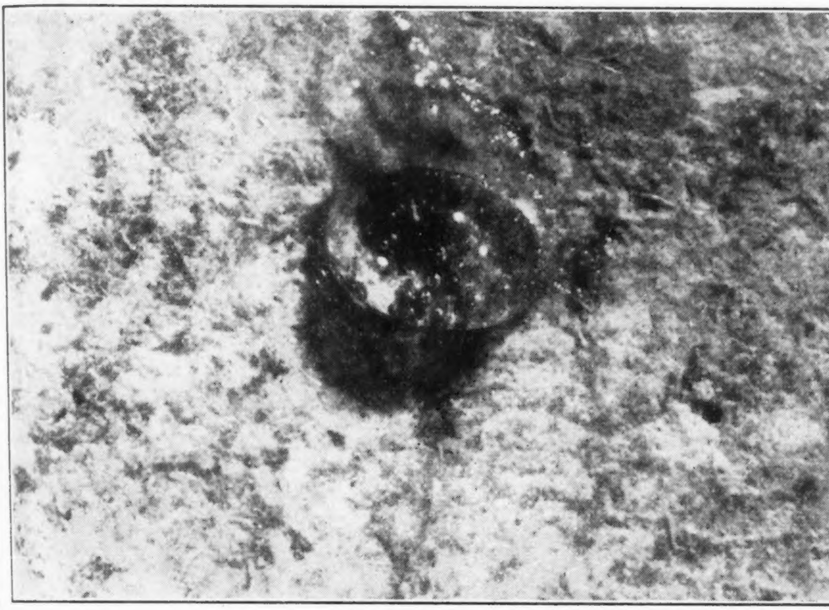

FIG. 12.—SODIUM SILICATE GEL IS FOUND IN THE PORE SPACES OF PARKER DAM CONCRETE

FIG. 13.—ANDESITIC AGGREGATE PARTICLES IN PARKER DAM CONCRETE ALTERED BY CHEMICAL REACTION WITH ALKALIES IN THE CEMENT

initiated should clarify the situation. In this connection, a comparison of the findings from the present investigations with the experiences described[21] by J. C. Pearson and G. F. Loughlin in 1923 with feldspar aggregates should prove extremely interesting.

Effect of Environmental Conditions on Concrete Deterioration at Parker Dam.—The drying conditions in the Parker Dam region are very severe, although observations on concrete structures in the arid regions indicate that such exposure is not particularly trying to concrete or conducive to unusual crack development. At Parker Dam, the cracking is most severe in the more massive concrete members and the abutment blocks at or near the roadway elevation. The cracking diminishes from the top of the dam downward, and there is practically no cracking at or near water levels. Also, at the top of the superstructure in the relatively thin wall, floor, and roof slabs of the gate-operating house, the cracking is insignificant.

For varying depths, to 30 ft, 10-in. by 20-in. cores have been drilled from the top of the spillway crest. The strength and Young's modulus values obtained with these cores are much higher than those previously reported[19] for cores taken at higher elevations. The average strength of the more recent 10-in. cores is 4,750 lb per sq in.; modulus, 3,375,000 lb per sq in.; and Poisson's ratio, 0.16. However, the recent cores from all depths profusely develop the characteristic exuding deposit of sodium silicate when stored in the presence of moisture. The gel-like deposit is found in the small pores within the concrete mass, and evidence of chemical attack upon particles of andesite and rhyolite aggregate can be seen—all as revealed by the earlier cores. The cementing matrix does not show the same degree of dissociation as previously observed, as might be expected from the higher strengths obtained with the later cores.

Improved Quality of Low-Alkali Cement.—The experiences of the Bureau of Reclamation in changing from relatively high-alkali cements to those of low alkali have developed interesting comparisons. The change from high to low alkalis with attendant improvement in other qualities has been made possible for certain Reclamation work through the splendid cooperation of the cement manufacturers in developing practical production methods. In most cases the alkali content has been reduced by controlling the selection and processing of raw materials and by increased burning temperatures. The combination of raw materials required for producing low-alkali cement results in a final product that is considerably higher in total silicates (C_2S and C_3S), with attendant lower C_4AF and C_3A, and the cement is more thoroughly burned. The typical examples in Table 7 illustrate the improved composition as a result of reducing the alkali content of certain low-heat cements. The sum of C_2S and C_3S (which is sometimes considered as one measure of the potential cementing value of a given cement) has been increased about 10%. In addition, the cementing efficiency, presumably, has been increased further as a result of higher burning temperatures.

In some mills it has been necessary for the producers to resort to the use of calcium chloride or other expedients in addition to raw-material selection in reducing the alkali content of their cement below the 0.60% limit. Calcium

[21] "An Interesting Case of Dangerous Aggregate," by J. C. Pearson and G. F. Loughlin, *Proceedings*, A. C. I., Vol. XIX, 1923, p. 142.

chloride apparently will effect a reduction in alkalis of at least 0.3% to 0.4%, depending upon manufacturing conditions. When it has been used, no evidence of chlorides has been found in the finished cement. In one case a heat-treating process of the clinker efficiently reduces the alkali content about

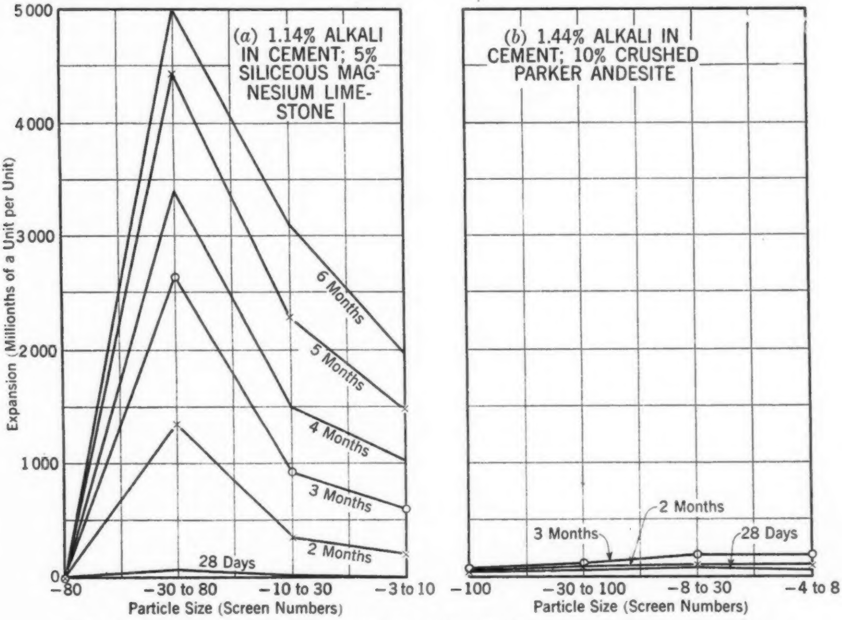

FIG. 14.—REACTION OF AGGREGATES TO THE MORTAR-BAR EXPANSION TEST AT 70° F

0.3%. In addition, the 7-day strength of this cement is increased 25%, for the same theoretical composition and fineness, by the heat treatment.

Experience to date indicates that the increased cost of producing cement that meets the 0.60% alkali limit may vary from practically nothing to 20 cents per bbl, depending upon the individual mill conditions.

TABLE 7.—EFFECT OF REDUCING THE ALKALI CONTENT OF CEMENT (PERCENTAGES)

Constituent	Mill A		Mill B		Mill C	
	Normal alkali	Low alkali	Normal alkali	Low alkali	Normal alkali	Low alkali
C_4AF	15	9	12	7	16	14
C_3A	6	5	6	6	6	4
C_2S	43	49	48	52	46	45
C_3S	29	32	29	30	25	32
Alkalis	1.2	0.4	0.9	0.4	1.3	0.4

Conclusion.—The engineering profession, the cement and aggregate industries, and the users of concrete are all indebted to Mr. Stanton. Without question, his discovery will ultimately lead to far-reaching improvement in concrete and better understanding of its ingredient materials.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF STATICALLY INDETERMINATE TRUSSED STRUCTURES BY SUCCESSIVE APPROXIMATIONS

Discussion

By L. E. Grinter, M. Am. Soc. C. E.

L. E. Grinter,[16] M. Am. Soc. C. E. (by letter).[16a]—This is the first paper on the analysis of trussed structures by successive approximations that has appealed to the writer as both interesting and practical. As mentioned by the author, several methods of successive approximations have been presented that complicated analysis instead of simplifying it. Actually, of course, the possibility of an important simplification does not exist for trussed structures because the usual procedure involving Maxwell's equations is not very complicated itself. There is a great difference between the difficulty involved in applying methods of deflection or work to a truss and to a rigid frame. In the first place, the truss is composed of members that are straight and are assumed to resist only direct stress. The application of virtual work for the computation of a deflection involves merely a simple summation of terms, and the calculations are readily tabulated. For a frame, integrations are always necessary and these may be complicated by variation of moment of inertia within each span, by distributed loadings, and even by a curved axis for one or more members. The other simplification that applies to trussed structures is that there are seldom more than three redundants to be considered, and, therefore, no more than three simultaneous equations need to be solved. The case of six or eight redundants in an indeterminate truss is almost limited to the case analyzed by the author as Example 2.

The derivation of basic relationships in the paper is made dependent upon Maxwell's Eqs. 3. Actually, one might write out Eqs. 5 directly as a statement of a self-evident fact. That is, the final stress in any redundant member a must be equal to the stress existing in that member when the other redundants are eliminated, plus the changes in stress that occur when these other redundants come into play in sequence.

Note.—This paper by O. T. Voodhigula, Jun. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Francis L. Castleman, Jr., Assoc. M. Am. Soc. C. E.; and April, 1941, by Messrs. Charles A. Ellis, and David J. Peery.

[16] Vice-Pres. and Dean, Graduate School, Illinois Inst. of Technology, Chicago, Ill.

[16a] Received by the Secretary April 3, 1941.

Hence, one may write at once without reference to sign, $X_a = s_a' + X_b K_{ab} + X_c K_{ac} \cdots$ (etc.); but it must not be overlooked that the terms s_a', K_{ab}, and K_{ac} demand the same deflection computations by virtual work required for the writing of Maxwell's equations.

The paper has been shortened to the point where certain fundamental considerations are not completely clarified. The computation of the deflection δ_{aa} is obtained by virtual work $\left(\sum \frac{S\,u\,L}{A\,E}\right)$ for six members only—those encompassed by a single panel. This is true because the virtual forces are unit stresses in the cut diagonal member, which are represented by a pair of equal and opposite forces having the same line of action as shown in Fig. 12(a). Since two such forces are in equilibrium, the end reactions of the truss are zero, and there are no stresses in other panels.

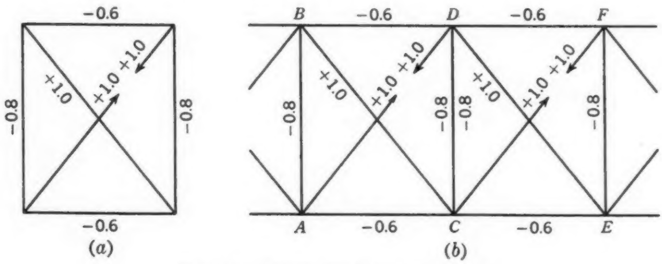

Fig. 12.—Action of Virtual Forces

The deflection δ_{ab} is computed even more simply than δ_{aa}. In Fig. 12(b), there are shown two adjacent panels of a truss with virtual forces (1 lb) acting in the redundant diagonals. The only member that receives stress from each of these virtual pairs of forces is the central vertical post $C\,D$. Since its stresses s and u are each -0.8 and its length is 32.0, the deflection δ_{ab} is computed as $0.8 \times 0.8 \times 32 \div A\,E$.

Many readers undoubtedly will question how the author simplified Eqs. 5 when they are applied to Example 2 in which there are eight redundants. One would assume that each of the eight equations should contain all such factors as K_{ab}, K_{ac}, K_{ad}, K_{ae}, K_{af}, K_{ag}, and K_{ah}, representing the interaction of the panel a with all other panels of the truss. Actually, all of these factors except K_{ab} are zero. The reason again may be found by reference to Fig. 12. For example, $K_{ac} = \delta_{ac} \div \delta_{aa}$. The deflection δ_{ac} would have to be computed by virtual work with virtual forces applied in the redundant diagonals of the panels a and c. In neither case would these redundant forces stress the same members and, therefore, the term $\frac{S\,u\,L}{A\,E}$ would be zero for each member. It is unfortunate when such matters as these are not explained in a technical paper since they save the reader much wasted time.

The writer has been greatly interested in the influence of errors in carry-over factors upon final results in any process of successive corrections. In Example 2, the carry-over factors are about 0.25 toward the end panels and roughly 0.20 in all other cases. In the latter instance, the actual variation of carry-over factors is from 0.165 to 0.218. In Table 6 the successive corrections of Table 3 are solved by use of the approximate carry-over factors 0.25 and 0.20. It is interesting that the maximum error for a redundant stress is 9% although

TABLE 6.—USE OF APPROXIMATE CARRY-OVER FACTORS FOR EXAMPLE 2

Description	Panel 1	Panel 2	Panel 3	Panel 4	Panel 5	Panel 6	Panel 7	Panel 8
1st $X = s'$	−1.75	−1.81	−1.71	−1.67	−1.67	+0.10	+5.42	+4.21
$K_{ab}\ X_b$, etc.	+0.45	+0.34	+0.33	+0.33	−0.02	−1.09	−0.84	
$K_{ba}\ X_a$, etc.		+0.26	+0.24	+0.23	+0.22	+0.30	+0.14	−1.18
2d X	−1.30	−1.21	−1.14	−1.11	−1.47	−0.69	+4.72	+3.03
$K_{ab}\ X_b$, etc.	+0.30	+0.23	+0.22	+0.29	+0.14	−0.94	−0.61	
$K_{ba}\ X_a$, etc.		+0.29	+0.26	+0.25	+0.23	+0.26	+0.12	−1.23
3d X	−1.45	−1.29	−1.23	−1.13	−1.30	−0.58	+4.93	+2.98
Correct X	−1.44	−1.21	−1.25	−1.23	−1.37	−0.53	+4.86	+2.93

the maximum error in the approximate carry-over factors was 18%. It is evident, therefore, that, whereas a small error in the solution of simultaneous equations is often very serious, small errors in the process of successive corrections tend to reduce themselves. One is often able to estimate carry-over factors with sufficient accuracy for preliminary studies. It must not be forgotten that the design of an indeterminate truss requires at least a preliminary analysis followed by a final analysis after the corrected areas have been chosen.

The method presented by the author is interesting and also useful if there are many redundants. The usual procedure by the direct solution of Maxwell's equations is so simple in itself that the use of successive corrections could offer little advantage for two or three redundants. Nevertheless, the author is to be complimented for presenting the most usable device yet described for analyzing indeterminate trusses by successive corrections.

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DISCUSSIONS

LABORATORY INVESTIGATIONS OF SOILS AT FLUSHING MEADOW PARK

Discussion

BY GORDON E. THOMAS, ASSOC. M. AM. SOC. C. E., AND M. N. SINACORI, JUN. AM. SOC. C. E.

GORDON E. THOMAS,[15] ASSOC. M. AM. SOC. C. E., AND M. N. SINACORI,[16] JUN. AM. SOC. C. E. (by letter).[16a]—In presenting a practical application of soil mechanics to foundation engineering the author is to be congratulated. The correlations between Atterberg limits and both shear and consolidation characteristics are indicative of more efficient methods to be used in soil investigations, especially from the preliminary and time-saving standpoints. In order to make these correlations of value, it is quite apparent that proper methods should be used in field sampling as well as in the simplest of laboratory tests.

With reference to the Atterberg limits, the writers wish to show, in passing, what correlations were found to exist in connection with a similar foundation investigation in another part of the United States. They particularly wish to show a further use of these Atterberg limits in borrow-pit studies for suitable impervious rolled-fill material for earth dams in which the material is composed of clay and silt mixtures.

The aforementioned studies were conducted on the foundation and borrow-pit areas of an earth dam situated in the Lower Tennessee River Valley about twenty-three miles from the river mouth. The earth structure is principally on the river flood plains, which are covered with a brown silty clay to a depth of from 10 to 30 ft. At a few locations layers of gray, carbonaceous, silty clay occur beneath the brown clay. In general, however, a 15-ft thickness of sand and clay underlies the top clay blanket, and this in turn overlies the fluvial valley fill consisting of sand and gravel. Samples of the foundation materials were taken with a drill rig using an "undisturbed" clay sampler, and also, where possible, by hand in cubic-foot boxes. Samples from the borrow areas were obtained by using hand augers of the bucket type.

The writers have found that for clays in the Lower Tennessee River Valley

NOTE.—This paper by Donald M. Burmister, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*.

[15] Associate Materials Engr., TVA, Gilbertsville, Ky.

[16] Asst. Materials Engr., TVA, Gilbertsville, Ky.

[16a] Received by the Secretary April 2, 1941.

the relationship between liquid limit and plasticity index is very close with only small variations from the mean curve. This fact made it possible to use only one of the Atterberg tests in the relations with the other characteristics. For example, there appears to be a relationship between plastic limit and the consolidation characteristics (namely, compression index and voids ratio), taken at 10,000 lb per sq ft. Because of the uniformity of the material and the narrow spread of characteristics, the number of samples tested was not as great as the number presented by Professor Burmister. The samples taken at the start of the investigation were 2 in. in diameter. The diameter was later changed to 3 in. The samples of larger diameter tend to give a more definite correlation. This is probably due to less disturbance in the material. It was also found that the accuracy of the correlation was influenced to a considerable degree by the presence of lenses and pockets of fine sand in some of the samples. The pressure-voids ratio curves obtained from these tests had initial voids ratios of from 0.7 to 0.9, thus placing the material in the range designated by Professor Burmister as silt in Fig. 3. In the case of shear characteristics, the

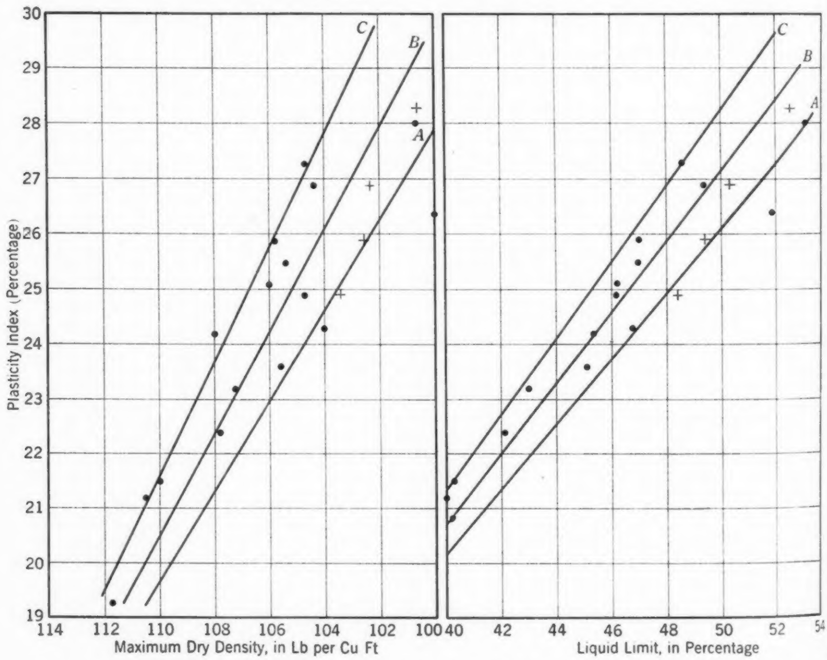

FIG. 9.—RELATION BETWEEN ATTERBERG LIMITS AND MAXIMUM DENSITY

writers found that the natural moisture content of the material had to be introduced as a third variable before a relationship of definite practical value could be realized.

These simple routine tests can be used to great advantage in obtaining information on the type of borrow material available for a rolled-fill section. The borrow areas for this particular earth dam are flat, long, and relatively shallow. The intrusion of sand layers, and high moisture contents at shallow depths, together with the necessity of leaving a prescribed thickness of blanket to minimize under-seepage of the dam, resulted in a small depth of borrow. All these factors made it necessary to investigate large areas, one of which was one-half square mile. Due to the stratification and variability of the soil,

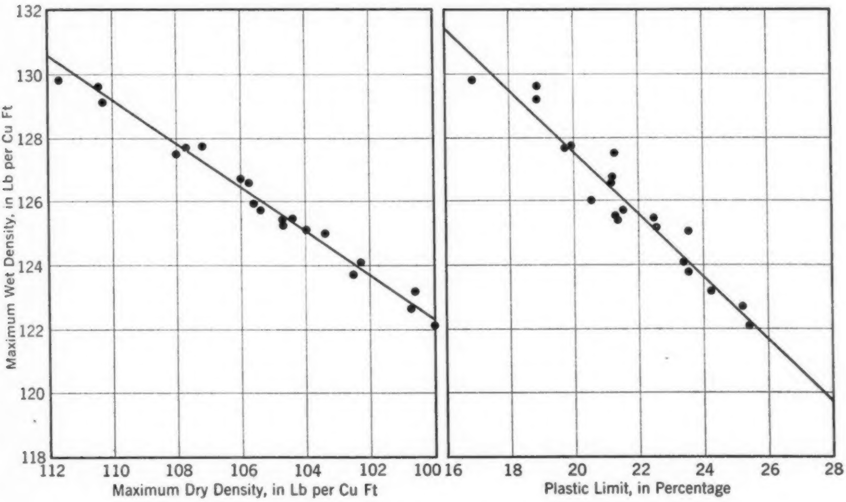

Fig. 10.—Relation Between Plastic Limits and Maximum Density

as well as the great area, a considerable number of compaction tests would have been required to cover this area completely. To save time and work, enough compaction tests were run to cover the various types of materials, as indicated by the pint jar samples from the auger borings. Then curves were drawn correlating liquid and plastic limits with compacted densities. From these original auger samples enough liquid and plastic limit tests were run to establish, more thoroughly, the compaction characteristics of the entire borrow area. Other general tests were run on typical borrow material to determine permeability, shear, and consolidation.

The liquid limit and plasticity index can be correlated very closely with the maximum density of the compaction curve by using the zone method. Fig. 9 shows the plots for a borrow pit on Twenty-Seven Mile Island, Kentucky. However, the plastic limit results, when plotted directly against the compacted maximum densities, show small variations from the mean curve, and are sufficiently accurate for correlating results. Consequently, these plastic limit tests alone were used as a rapid means of estimating maximum densities (Fig. 10). From the foregoing relationships all borrow-pit materials were classified very closely.

In dealing with borrow materials, the soil is thoroughly mixed when prepared for compaction, and any local variation in the sample is eliminated. In general it may be stated that Atterberg limits can be more closely correlated with borrow materials than with foundation soils taken in the natural state.

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DISCUSSIONS

CONCRETE IN SEA WATER: A REVISED VIEWPOINT NEEDED

Discussion

By Messrs. Lester C. Hammond, Ladis H. Csanyi, and G. M. Williams

Lester C. Hammond,[16] M. Am. Soc. C. E. (by letter).[16a]—In his "Conclusions" the author states that "The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement."

Although he agrees heartily with this statement, the writer cannot agree that the lack of evidence Mr. Hadley has observed, of sulfate attack in concrete marine structures, warrants him in assuming that sulfates in sea water do not reduce the life of the concrete. The writer observed concrete in sea water that was constructed in 1911, which in 1924, after 13 years of service, showed no signs of deterioration; yet the same concrete, when inspected in 1934, showed signs of surface disintegration.

The concrete viaducts of the East River Drive in New York City were built in sea water. The East River is a tidal estuary connecting the Long Island Sound and New York Bay. During construction, Ladis H. Csanyi, M. Am. Soc. C. E., made an extensive series of tests on twenty-nine concrete cylinders, of which all but nine were samples of the actual concrete used in the structures. These tests consisted of the complete submersion of the samples in a 43% solution of Epsom salts, which is equivalent to a 20% solution of anhydrous magnesium sulfate. This solution is estimated to be about fifty times as strong as the sulfate concentration in the waters of the East River. The test was limited to purely chemical action. The result of these tests proved that, within 92 days, imperfect concrete would show surface attack, and that even the best concrete would not withstand this chemical action after 189 days. Many of the tests extended over a period of 250 days. This proves to the writer that the sulfates in sea water have some chemical action on any concrete.

Note.—This paper by Homer M. Hadley, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Thomas E. Stanton, M. Am. Soc. C. E.; and April, 1941, by Messrs. W. F. Way, and Glenn S. Paxson.

[16] Chf. Engr., Borough of Manhattan, New York, N. Y.

[16a] Received by the Secretary March 28, 1941.

The visible effects on good concrete are considerably less than on poor concrete. Since the tricalcium aluminate (C_3A) content of the cement has a bearing on the resistance of the concrete to sulfate action, it is advisable to use sulfate-resistant cement. The tests confirmed this view and it was noted that concrete having a computed C_3A percentage between 6% and 7% showed best results.

It is not proper to assume that because the deterioration of the concrete observed in Pacific Coast waters can be largely explained by the fact that the concrete was improperly constructed, that the sulfates in the sea water did not contribute to this deterioration. In 1935, Rudolph J. Wig and Lewis R. Ferguson,[17] Members, Am. Soc. C. E., demonstrated that chemical action was noted all along the Pacific Coast. Furthermore, the report of Thomas E. Stanton,[18] M. Am. Soc. C. E., in 1938, indicates that chemical action has been cause for concern along the California Coast.

The reasons for concrete failure in sea water, stated in their relative importance, are: Improper placing, and forms; improper grading of aggregates; insufficient cement; excess of water; improper character of aggregates; and too much C_3A in the cement.

The causes are: Scouring action, frost action, and the chemical action of the sulfates in the water.

With the foregoing possibilities of failure, it is reasonable to claim that the chemical action of the sea water is not the most important cause of its failure; but it seems unreasonable to believe that it is not a contributing factor.

LADIS H. CSANYI,[19] M. AM. SOC. C. E. (by letter).[19a]—During the spring of 1939, nineteen concrete structures situated in the harbor of New York City were examined, and all pertinent information concerning the design, mixing, and placing of the concrete was compiled and analyzed. The results of this study confirm the findings of Mr. Hadley, in that a dense, uniform concrete properly placed is essential wherever sea-water exposure is encountered. Only three of the structures inspected showed a resistance to disintegration for periods of twelve years. Of these three, two utilized cements, approaching a moderate sulfate-resistant cement, by containing a computed C_3A-value of 9%. The other, in service for thirty years, yielded no information concerning the source or composition of the cement.

In an inspection such as this and the one conducted by Mr. Hadley, the area open to visual examination is wholly above dead low water, wherein the differentiation of physical and chemical reactions is extremely difficult, due to the violent effect of mechanical action. The part below this area, which is totally and continuously submerged, where mechanical and physical action is greatly reduced, and where chemical action is increased, cannot be examined without diving gear. Thus, no conclusive evidence could be obtained by this

[17] *Engineering News-Record*, Vol. 79, Nos. 12, 14, 15, 16, and 17.

[18] "Testing Cement Mortars in Sea Water," by Thomas E. Stanton, *Engineering News-Record*, March 17, 1938, p. 400; also "Expansion of Concrete Through Reaction Between Cement and Aggregate," by Thomas E. Stanton, *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781.

[19] Asst. Engr., Engr. in Charge, Municipal Asphalt Plant, Dept. of Borough Works of Manhattan, New York, N. Y.

[19a] Received by the Secretary April 3, 1941.

examination to support, satisfactorily, the contention that cement is not a factor in the resistance of concrete in sea water.

In order to obtain further information concerning the behavior of concrete in that part totally and continually submerged, a series of tests was undertaken by the writer.[20]

The test consisted of subjecting twenty-nine standard concrete cylinders to total continuous submersion in a solution of Epsom salts. During the test the concentration of the solution averaged 27.2° (Baumé), or a 43% solution, which is slightly below saturation. Magnesium sulfate was selected as the test medium because: (1) Its concentration and composition can be controlled easily; (2) it is the main corrosive agent found in sea water; and (3) it is used in checking the soundness of aggregates used in concrete. The concentration was set high primarily to accelerate the test, and secondarily because the soundness test for aggregates used a saturated solution. "What is sauce for the goose is sauce for the gander." The concrete used for the cylinders was obtained in all except eight instances directly from the chutes leading into the forms of the bulkhead walls being constructed for the East River Drive (New York City) in order to provide concrete actually used in the structure for the test. The cements used were all of the sulfate-resistant type, varying in computed C_3A-content from 4.1% to 6.6%. The mixes tested were 1 : 1.8 : 3.2 and 1 : 2.9 : 5.1. In order to check cements of higher C_3A-content, the remaining eight specimens were prepared in the laboratory, having a mix of 1 : 2.2 : 3.9 and using cements having computed C_3A-contents of 9% and 16%.

The results of the test are briefly as follows:

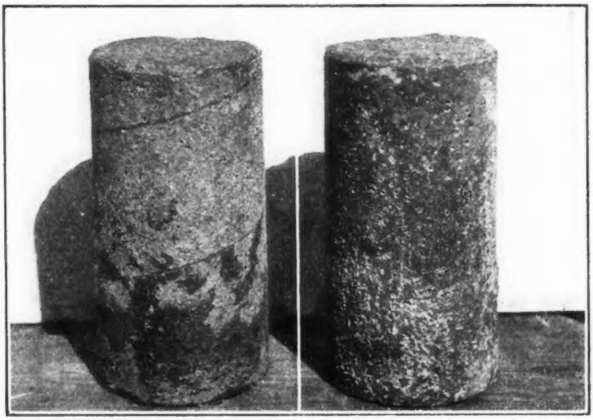
(*1*) *Effect of Sulfate Action on Compressive Strength.*—The compressive strength of the concrete under the test increased generally to 30% above the standard 28-day check test after extended immersion in spite of surface disintegration. The increase stopped and decrease started upon approximately 50% of surface disintegration when further increase in strength was offset by disintegration.

(*2*) *Effect of Curing on Rate of Disintegration.*—Samples cured for 3 days (1 day water, 2 days wet sand) showed a marked difference in the form of surface disintegration from those cured for 25 days (1 day water, 24 days wet sand). In the 3-day cure (Fig. 8(*a*)), the attack penetrated the grout or carbonate skin and progressed beneath it, flaking off large sections at a time, and thus rapidly exposing the aggregates of the concrete. In the 25-day cure (Fig. 8(*b*)), disintegration was isolated to the points of attack, giving the concrete a blotchy, pimply appearance. In this case exposure of the aggregates was limited and progress of disintegration materially curtailed.

(*3*) *Relation of Design of Concrete Mixture to Rate of Disintegration.*—In every case the richer mixes possessing the greater density showed better resistance for the same type of cement. Mixes of similar density with cement of 16% computed C_3A-content showed less resistance than one having a 9% computed C_3A-content.

[20] "Resistance of Concrete to Magnesium Sulphate Solution," by Ladis H. Csanyi, with the cooperation of the Dept. of Borough Works of Manhattan, New York, N. Y., submitted to the Polytechnic Institute of Brooklyn, N. Y., in 1940, in partial fulfilment of the requirements for the degree of Master of Civil Engineering.

(*4*) *The Influence of Cements Upon the Rate of Disintegration.*—The comparison based on the computed C_3A-content of the cement clearly showed that reduction of C_3A-content from 16% to 9% to 6% materially improved the resistance of the concrete. Reduction of C_3A-content below 6% to 4.5% and 4.1% showed material loss of resistance. Therefore, a range between 6% and



(*a*) Three-Day Cure (*b*) Twenty-Five-Day Cure
FIG. 8.—CONCRETE CYLINDERS AFTER BEING IMMERSED 92 DAYS

7% computed C_3A-content of the cement is indicated as yielding the maximum sulfate resistance to the concrete as far as the effect of the cement is concerned.

These studies, therefore, further confirm the essential need of good, dense, uniform concrete, carefully and properly placed and adequately cured, if good sea-water resistance is to be attained. They also refute Mr. Hadley's contention, and confirm the work and belief of others,[21,22,23] that cement is important in imparting some resistance to the concrete when subjected to sea water.

The writer's studies revealed two questions that concern engineers: "What is sea water?" and "What does it contain?" In harbors it may vary from brackish water to high concentrations of sewage frequently containing corrosive industrial wastes. Therefore, let the engineer study the sea water in which he must place his structure, and provide adequate protection and properly designed mixes placed with especial care when the sea water requires it.

G. M. WILLIAMS,[24] M. AM. SOC. C. E. (by letter).[24a]—The conclusion in Mr. Hadley's paper that there is no evidence of attack or disintegration due to sulfate action could be accepted more readily if chemical analyses of concretes

[21] "Testing Cement Mortars in Sea Water," by Thomas E. Stanton, M. Am. Soc. C. E., *Engineering News-Record*, March 17, 1938, p. 400; also "Expansion of Concrete Through Reaction Between Cement and Aggregate," by the same author, *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781.

[22] "Influence of Composition on Volume Constancy and Salt Resistance of Portland Cement Pastes," by R. H. Bogue, William Lerch, and W. C. Taylor, 1934.

[23] "The Chemistry of Cement and Concrete," by F. M. Lea and C. H. Desch, *24 Concrete Manual*, U. S. Dept. of the Interior, 1938.

[24] Prof., Civ. Engr., Univ. of Saskatchewan, Saskatoon, Saskatchewan, Canada.

[24a] Received by the Secretary April 17, 1941.

were offered in support. Although developed porosity or disruption may not be apparent, this is not necessarily good evidence that deterioration due to sulfate action is not in progress. Rich concrete mixtures may fail by cracking without prior indication of surface swelling, spalling, or disruptions which are characteristic of failures of medium or lean concretes.

Chemical analyses of small samples taken at several points in a structure such as between low and high tide, parts exposed only to spray, and parts never in contact with sea water should indicate the extent of sulfate action, if any, as well as measure its rate of progress.

Concrete of low permeability is desirable for permanence and resistance to weathering or possible sulfate action, but such concrete is generally not one of greatest density. Neat cement, at one extreme, is impervious but has high porosity and low density, whereas a lean concrete may be very pervious but low in porosity and high in density. Both are absorptive. The terms impermeability and density are antonyms rather than synonyms as used to designate concrete qualities.

That concrete in sea water sometimes deteriorates is admitted. Other investigators have attributed this at times to the action of sulfates. The author, who comments upon the somewhat unusual dearth of supporting laboratory data, believes that no sulfate action has occurred, basing his conclusion upon the results of visual inspection and the absence of either developed porosity or swollen disruption of the mass which he assumes to be the characteristic form and manifestation of sulfate of magnesium attack. Since sulfate content of concrete can be determined so readily by chemical analyses of easily obtained samples, it would seem that valid conclusions can be drawn only after such data are obtained.

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DISCUSSIONS

DESIGN OF ACCELERATION AND DECELERATION LANES

Discussion

By Messrs. H. F. Holley, D. W. Loutzenheiser, Hawley S. Simpson, and Milton Harris

H. F. Holley,[7] M. Am. Soc. C. E. (by letter).[7a]—The design of acceleration and deceleration lanes as presented in this paper clarifies a theory that engineers engaged in highway design have been applying in a more or less rule-of-thumb manner for many years.

Acceleration and deceleration lanes are fast becoming the rule rather than the exception in modern highway construction, especially in the design of motorways in metropolitan areas. However, the opportunity rarely presents itself for the application of theory in laying out motorway entrances and exits in congested metropolitan areas. Land values and topographic controls generally restrict the length of the acceleration and deceleration lanes and result in their construction in lengths far less than theoretically indicated.

The author's typical plan (Fig. 7) calls for 1,665 ft of entrance and exit lanes, plus 258 ft of treated shoulder on a highway designed for 50 miles per hr. This demands an extra lane for a distance of 1,923 ft, or 0.36 mile. Even in rural territory it is unusual to find sections of highway along which entrances and exits can be spaced as far apart as one third of a mile. Furthermore, for rural highways, designed (and actual) speed is nearer 70 than 50 miles per hr. Entrance-exit treatment as proposed by Mr. Mitchell on most 60-mile rural highways would call for a continuous extra lane. It was this condition that led to the practice of establishing "service roads" on either side of high-speed rural highways in territory subject to development—first into residential, then small business, and finally into "shoestring" business use.

In the writer's judgment, Mr. Mitchell has been ultraconservative in his requirements for motor deceleration and braking distances. Extensive observations on driver habits in the Los Angeles (Calif.) metropolitan area indicate that deceleration by engine compression is seldom practiced. California drivers

Note.—This paper by Adolphus Mitchell, Assoc. M. Am. Soc. C. E., was published in March, 1941, *Proceedings*.

[7] Asst. Chf. Engr., Automobile Club of Southern California, Los Angeles, Calif.

[7a] Received by the Secretary March 31, 1941.

tend to approach even a boulevard "stop" intersection under power, and apply their brakes immediately after releasing the throttle. Upon approaching a signalized intersection, the tendency is to speed up in the hope of finding the signal on the "go" upon arrival at the intersection.

The "comfortable braking distance," shown graphically in Fig. 4, indicates about 50 ft to stop from a speed of 20 miles per hr, 135 ft from a speed of 30 miles per hr, and 280 ft from a speed of 40 miles per hr. Table 1 shows the

TABLE 1.—BRAKING DISTANCES, IN FEET

Agency	Year	Speed in Miles per Hr		
		20	30	40
National Safety Council	1935	21	47	84
Aetna Casualty and Surety Company	1935	22	49.5	88
"Traffic Engineer and the Police"; consulting bureau	1938	26	57	101
National Bureau of Standards	1935	28	62	109
Automobile Club of Southern California	1930	22.2	50	89

braking distances determined from experiments by the agencies listed. It is admitted that these braking distances are more in the nature of emergency braking, rather than "comfortable" braking distances. Also, they do not include "reaction-time distance" which, using the accepted reaction time of 0.75 sec, would add 22, 33, and 44 ft, respectively, to the braking distances shown at 20, 30, and 40 miles per hr.

Mr. Mitchell's paper provides an objective toward which designing engineers should strive. However, in metropolitan motorway design the ideal entrance and exit ramp rarely can be achieved.

The Arroyo Seco Parkway, a motorway extending from the heart of metropolitan Los Angeles to the neighboring City of Pasadena, completed in 1940, involved twenty-two vehicular crossings in a distance of 6 miles. It was impossible, due to constriction caused by the Arroyo Seco Channel and other topographic features, to build conventional "clover leafs" at points of vehicular interchange. It was also impossible, due to the large number of intersecting streets, to design entrance and exit lanes of desired length. However, nonstop motorway features were adhered to, the design calling for two 35-ft roadways separated by a planting strip, no intersections at grade, no left turns, and the prohibition of ingress and egress from abutting property. To compensate for the lack of the desired length of the acceleration and deceleration lanes, a "compressed clover leaf" was designed at most interchange crossings. Generally (see Fig. 14), the exit and entrance ramps at intersecting streets are approximately parallel to the motorway; and, due to the fact that the motorway roadway is depressed, the exit ramps are on ascending grades, whereas the entrance ramps are on descending grades, permitting relatively fast deceleration and acceleration, with the path of the entering or leaving car roughly parallel to the main roadway. In spite of the fact that entrance and exit lanes are much shorter than indicated as desirable under Mr. Mitchell's theory, traffic using the Arroyo Seco Parkway (which averages about 40,000 vehicles per day) experiences little difficulty in entering or leaving the motorway, and there has been a very favorable accident experience at interchange points.

In the case of an elevated motorway, with intersecting streets crossing under the structure, an unfavorable deceleration and acceleration situation would be

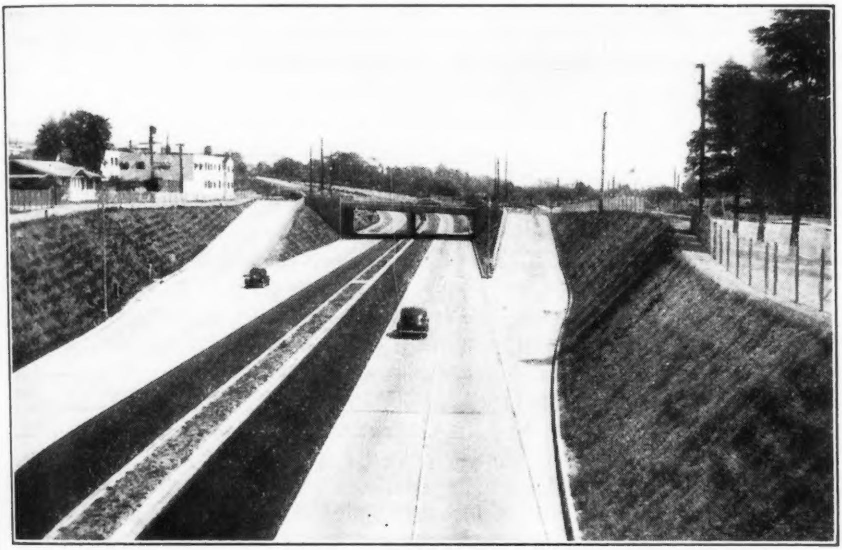

FIG. 14.—THE ARROYO SECO PARKWAY AT FAIR OAKS AVENUE, SHOWING TYPICAL ENTRANCE AND EXIT RAMPS OF COMPRESSED CLOVER-LEAF DESIGN

encountered at interchange points, and longer exit and entrance lanes should be provided.

Mr. Mitchell is to be commended for reducing to scientific form a matter that highway engineers have been inclined to treat in a haphazard manner.

D. W. LOUTZENHEISER,[8] JUN. AM. SOC. C. E. (by letter).[8a]—The subject of speed-change lanes is an important phase of highway design, to which considerable research and study should be applied. Although the author infers that his paper is the initial attempt to rationalize design of speed-change lanes, it appears to be an academic discussion, somewhat extended in detail, of a design approach made in a paper[9] published elsewhere, in 1940. The details of the bases for design of the high-type speed-change lanes on the connections to the Pennsylvania Turnpike also have appeared in varying degree in engineering publications.

As is the case for many phases of geometric highway design for which the preparation of "standards" has not yet been possible, the design of speed-change lanes involves evaluation of the intangible "driver behaviors" to evolve a

[8] Associate Highway Engr., Div. of Design, Public Roads Administration, Washington, D. C.

[8a] Received by the Secretary April 9, 1941.

[9] "A Policy on Intersections at Grade," Am. Assoc. of State Highway Officials, 1940.

practical solution. The result must be simple and perfectly understandable to those using it, but above all it must be so dimensioned and located that drivers can, and will, automatically use it as the designer intended. The approach for design must be of the type described by Mr. Mitchell, and elsewhere previously, but the entire problem is of such nature that the academic detail and precision of values in this paper seem unnecessarily extensive. The basic data upon which the various factors are assumed or calculated have not yet been evaluated to the degree that they are applied. As one example, the degree of accuracy inferred in the mathematical extension of the acceleration and deceleration rates (Figs. 3 and 8) for ascending and descending grades is not at all consistent with the limited number of vehicles, drivers, and conditions of test from which the rates were determined.

Speed-change lanes generally will fall in the class of emergency pavement area—they will be used as some designer predicts only by the occasional vehicle that is forced into that sequence of actions and control by other streams of traffic. For the most part, they are widened pavements. In the absence of other traffic, drivers will enter or leave the outer lane with considerable variance in position and speed, as found necessary at the particular instant. One large field of application of speed-change lanes is at intersections on two-lane and three-lane roads. As shown in Figs. 1 and 7, Mr. Mitchell obviously excludes these cases in order to introduce the insulation strip. The writer feels that in most instances, even along express highways, the flow of traffic will not be facilitated to the extent possible by considering long insulation strips as an essential part of the design. Their chief function is to prevent improper left turns, but as far as use of the speed-change areas is concerned they are better omitted or limited to short lengths. It is stated that the function of the insulation strip is to eliminate points of conflict. The introduction of such strips adds an island point that may be more hazardous than beneficial.

In reference to Fig. 6, Mr. Mitchell states that operators will not drive as fast on narrow pavement as on broad pavement, citing data reported by Messrs. Thompson and Hebden as proof.[5] This is questionable usage—conditions other than pavement width usually determine speed of travel on any highway. The important point is that faster traffic generally will drive a greater distance from the edge of pavement than will slower traffic on the same pavement. The placement data quoted[5] were observations on the average position of a "critical vehicle"—that is, placement on a two-lane road as the vehicle was passed by another vehicle. This may or may not be representative of placement on a one-way road, or on roads of other widths; but it does not prove that narrower pavements result in lower speed operations. A study in Missouri[10] reported in 1940 showed that high-speed traffic traveled just as fast on an 18-ft as on a 20-ft pavement, with average speeds of 53 miles per hr, and a top speed of 88 miles per hr in each case. With the available placement data as a basis there seems to be little justification for any variation in the width of speed-change lane, provided it is wider than a minimum of 11 or 12 ft. A desirable width for

[5] "A Study of the Passing of Vehicles on Highways," by J. T. Thompson and Norman Hebden, *Public Roads*, U. S. Dept. of Agriculture, Bureau of Public Roads, 1937, pp. 121–137.

[10] "Speed of Motor Vehicles on Missouri Highways," Missouri State Highway Dept., 1940.

a one-way lane is 14 ft, barely permitting passage at slow speed past a vehicle stalled on the lane. In the case of the deceleration lane it is fundamentally incorrect to provide narrower widths as the sharp radius curve (50 ft) is approached. The lane width around this curve should be 14 to 17 ft as a minimum.

The evaluation of weaving distance might be clearer if also expressed as a time interval. The speed-distance values calculated from a friction factor of 0.16 approximate that for about a 3-sec interval, which method of presentation is not as suggestive of great refinement in the accuracy of the calculation bases.

A comprehensive study of the speed, placement, driver behavior, and all other pertinent factors in the operation of vehicles on existing speed-change lanes is sorely needed as a basis for proper design of such lanes. Until such data are secured any great refinement for design details based largely on assumptions for the various factors should be considered only as of academic value. Even a short length of added lane will facilitate the flow of vehicles turning sharp curves. Although the benefits of longer lengths of added lane may be anticipated (and doubtless correctly so), it cannot yet be implied or stated that they are proven. It is to be hoped that this paper will focus attention on the need for field data to be used as a basis for proper design of speed-change lanes.

Hawley S. Simpson,[11] M. Am. Soc. C. E. (by letter).[11a]—Mr. Mitchell has opened a very interesting subject, and one that has been largely ignored in the past. Entrances and exits to and from high-speed highways have been mainly designed by the "catch-as-catch-can" method. As more express highways in rural districts are built, greater attention must be given to reducing the effect of conflicts caused by vehicles entering and leaving the highway.

With the general principle of the deceleration lane the writer is thoroughly in accord. Regardless of whether traffic is light, or whether it is heavy, vehicles about to leave an express highway may be followed closely by others moving at the same speed. To move the turning vehicle out of high-speed traffic without slowing following vehicles and to reduce the probability of rear-end collisions are the purposes of the deceleration lane. The design proposed by Mr. Mitchell fulfils these requirements quite adequately. The only valid criticism is that it may be too adequate. The author has treated the motorist with greater consideration than the economics of the situation warrants. With the little that is known about habits of drivers it seems quite possible that they would react as favorably to a design that does not involve the rather long distances proposed by Mr. Mitchell. From his own driving experience, the writer sees no reason why such a lane should not be designed to permit deceleration, without braking application, to 40 miles per hr instead of 30 miles per hr and then require a much harder brake application than that proposed by Mr. Mitchell to reduce the speed of the vehicle to a safe turning speed.

The deceleration lane designed by Mr. Mitchell will give the impression of

[11] Research Engr., Am. Transit Assn., New York, N. Y.

[11a] Received by the Secretary April 10, 1941.

extreme length, and many, if not most, drivers will continue at "cruising" speed for some distance down the deceleration lane. Then they will coast to about 40 miles per hr, and make a much harder brake application than that required by the use of a coefficient of friction of 0.16.

The design of an acceleration lane, however, is in a decidedly different category. Nevertheless, Mr. Mitchell has applied practically the same principles in reverse order. Here the vehicle is assumed to enter the acceleration lane at a relatively low speed and accelerate exactly to cruising speed. The driver reaches this speed at precisely the point where he is expected to begin "weaving" into the cruising lane. At this exact time he must find a gap in traffic sufficient to accommodate his vehicle, and one that will provide clearance space between preceding and following cars, or he must abandon the maneuver and make a hard brake application in the emergency stopping space. There he will await an opening in traffic and enter just as if no acceleration lane had been provided.

Whether such a lane is proper is a moot question. Consider the two extremes of traffic—(1) extremely light, and (2) extremely heavy. In the first situation, there is a high probability that the required opening will be available upon arrival at the weaving section. If this is so, why is an acceleration lane necessary? Would it not be just as feasible and much less expensive to require the vehicle to come to a full stop at a normal right-angle intersection and await an opportunity to enter which, theoretically, would present itself almost immediately?

At the other extreme, when traffic on the main highway is very dense, the probability that the driver, having reached cruising speed, will be able to enter the traffic stream at the exact moment he arrives will be so remote that a very large percentage of the entering vehicles will be forced to use the emergency stopping area. Thus, they will be required to enter the traffic stream exactly as if there had been no acceleration lane; in other words, as if the intersection had been designed as a normal right-angle entrance.

For traffic volumes between these two extremes the condition will partake of the nature of either one or the other of the two cases just described.

Thus, it seems more logical to forego the construction of an acceleration lane, and to require traffic to stop at the entrance to a high-speed highway and await an opportunity to enter without interfering with other vehicles.

From the standpoint of accidents it seems likely that this solution is preferable to the construction of a deceleration lane. A driver in a car moving on to a main highway, and at the same speed as the traffic on it, is in a most difficult position to see and appraise the speed and position of vehicles at his left rear. His chances of being able to enter that traffic at the precise point where there happens to be an opening is so slight as to insure a marked increase in traffic accidents. Construction of such lanes should proceed cautiously until an opportunity has been afforded to learn the outcome more accurately.

Commenting upon the accident experience on the Merritt Parkway in Connecticut under the heading "Justification," Mr. Mitchell states, from rather inconclusive evidence, that: "there remains little doubt as to the location of a sizable percentage of the rear-end collisions * * *." The writer doubts if this conclusion is warranted by the facts and strongly urges a more intensive accident study of highways of this kind before exact conclusions are attempted. Many hours of driving on the Merritt Parkway have led the writer to conclude that there is a substantial potential hazard in the inadequacy of the deceleration lanes, and relatively little hazard at entrance points which, though providing a short length of additional roadway width, partake more nearly of conventional design.

MILTON HARRIS,[12] ASSOC. M. AM. SOC. C. E. (by letter).[12a]—The "Design of Acceleration and Deceleration Lanes" has been treated admirably by Mr. Mitchell in segregating the design requirements into their component parts. This approach allows one to weigh the relative importance of each element and to give consideration to the data upon which the design is founded.

Following this line of thought, there seems to be little to criticize in application of the tables and curves if they are assumed to be inherently correct and applicable to this problem. The only exception in application seems to lie in advocating an improved shoulder for braking distance beyond the exit from the acceleration lane. It would seem that a car using the acceleration lane could see parallel moving vehicles and so adjust its speed, while still within the lane, to permit it to mix without stoppage when the exit was reached. This presumes that the insulating strip is a low curb or fin across which a motorist can see. An insulating strip is an excellent device to channelize traffic.

One element of design that does not seem to be included in the paper is the relation of volume to lane length and width. This is of particular importance when designing acceleration and deceleration lanes for large turning movements and storage where at least two lanes are required to handle the volume efficiently. The problem grows still more complicated when consideration is given to the necessity of crossing deceleration volumes with the acceleration stream at an intersection and signalization is introduced.

One is led to believe that traffic engineers are prone to accept formulas and experimental data without due regard to their practical application, both from an economic as well as a traffic standpoint. The data presented in this paper in the form of tables and curves have been collected over a period of time under conditions that were hardly similar to those presented. Although it cannot be said that such data are not applicable to this design problem, the fact still remains that much research must be conducted to ascertain if fundamental data are applicable to a variety of conditions.

With an unlimited laboratory at the "front door," it is suggested that new designs for traffic use be subjected to actual test under a variety of conditions before adoption. In the case of acceleration and deceleration lanes, an improved shoulder adjacent to an intersecting road could be used and sandbags placed to simulate the insulating island. The speed, volume, and wheel

[12] Associate Highway Engr., State Div. of Highways, Sacramento, Calif.

[12a] Received by the Secretary April 11, 1941.

placement patterns could be collected and analyzed for various lengths of lane under similar conditions, thus proving or disproving the theoretical deductions in so far as that one locality was concerned.

Corrections for *Transactions:* March, 1941, *Proceedings*, p. 377, change line 4 from the bottom to read, "* * * desirable braking distance, as given by interpolation in Fig. 9(*b*), is 280 ft. Thus it is determined * * *"; change X^2 to V^2 in Eq. 9; line 6, p. 380, change "$g = 322$" to "$g = 32.2$"; in Fig. 11 change "S" to "s"; and in Eq. 12, divide the quantity $g\ G$ by 100.

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DISCUSSIONS

VALUE OF PUBLIC WORKS

Discussion

BY MESSRS. UEL STEPHENS, WILLIAM J. WILGUS, BERNARD L. WEINER, ALBERT ED. SCHEIBLE, H. B. COOLEY, AND PHILIP W. HENRY

UEL STEPHENS,[5] M. AM. SOC. C. E. (by letter).[5a]—The subject which Major Hallihan has so ably presented is most appropriate at this time when the entire attention of the United States is being centered on its defense requirements. Now is the proper time for the engineering profession to formulate plans for "carrying on" after the flush of defense construction subsides, and buying begins to decline, and finally reduces to a minimum.

It is a well-established fact that every period of unprecedented construction and spending activity must be followed by a period of below-normal activity that usually will exceed both in duration and intensity the preceding period of above-normal activity. It appears that the amplitude or distance between the crest of the wave of prosperity and the bottom of the valley of the depression increases in direct proportion to the degree of industrialization. The capability of industry and small political subdivisions to combat the periods of depression, and to keep within reasonable bounds the extent of expansions during periods of abnormal development, appears to become less and less, and the tendency to look to the federal government for the solution increases proportionately. These facts being accepted, it then behooves the federal government through its proper agencies to begin at once, and rapidly develop, a vast plan that will contain a large reservoir of worth-while public projects that can be placed under construction rapidly and economically in accordance with adequate engineering designs.

The engineers who experienced the many phases of public building from the Emergency Relief and Construction Act of 1932 to the defense construction programs of 1940 and 1941 should realize, better than members of any other profession, the dire needs for adequate and proper planning before the de-

NOTE.—This paper by J. P. Hallihan, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1941, by Clarence W. Post, M. Am. Soc. C. E.

[5] Supt., WPA Projects, State Highway Dept., Austin, Tex.

[5a] Received by the Secretary March 24, 1941.

pression comes. Their experience in rushing into great construction programs in 1933 and 1934 clearly demonstrates the waste and inefficiency that cannot be avoided unless some one central agency has planned and can be given the necessary authority to execute some well-developed and properly coordinated plan for construction projects of all types. Probably one of the most important duties of such a federal planning agency would be the coordination of the activities of various agencies. Competition between governmental agencies, such as existed between the PWA and the WPA and now (1941) is occurring between the WPA and the PRA (Public Roads Administration), should be made impossible.

If the federal government, through its National Resources Planning Board, could provide funds in the form of grants to municipalities to pay the engineering cost, in part or in whole, for the preparation of preliminary plans and specifications covering such projects as could be undertaken for construction, there would be an incentive for private engineers to plan a program that could be constructed with little delay when the necessity arose. Such an organization must be local to the extent that the study of prevailing local conditions can be made. Such work could not be done successfully by a central organization in Washington, D. C., for the reason that local conditions could not be comprehended readily. By decentralizing and placing complete responsibility in the hands of state organizations, the local engineers would be more prone to plan and study projects if they thought that final consideration could be given locally rather than being reserved by a central office—some located great distances away.

In conclusion, Major Hallihan is to be complimented for the very able paper he has presented. Based on experience and on the records that have been kept, it is evident that the present is the proper time to begin planning for the next depression, which is sure to follow in the wake of the present unprecedented construction program.

William J. Wilgus,[6] Hon. M. Am. Soc. C. E. (by letter).[6a]—In these days of peril it is noteworthy that Major Hallihan's thoughtful paper should have appeared simultaneously with the report[7] of the National Resources Planning Board in favor of the "storing up [of] a reservoir of non-defense public works which can be loosed when the pace of rearmament slackens." There would seem to be no room for argument that the United States now should prepare for the defense of its institutions from attacks from within as well as from without. It is not difficult to visualize what will occur if countless of its citizens are to be permitted again to suffer from unemployment when the war boom ends. In a word, the two kinds of preparedness must go hand in hand if wisdom is to prevail, even though many may feel that preparedness for internal peace should "take a back seat" while the nation is engaged in preparedness for external war. To win in one and not in the other will spell ruin.

[6] Weathersfield, Ascutney P. O., Vt.

[6a] Received by the Secretary March 26, 1941.

[7] "Development of Resources and Stabilization of Employment in the United States," National Resources Planning Board, January, 1941.

It is widely urged that the furtherance of projects "which are either economically non-productive or of a sort which displaces private enterprise in its own legitimate field" will mark "a plunge headlong into state socialism and totalitarian government." [8] There is indeed risk of this in a world afire; but if large-scale employment ceases, and with it the buying power of multitudes thrown out of work, what will enable free enterprise to take up the slack any more than in 1933? Idle money means idle men—the death not only of free enterprise but of all else that Americans hold dear.

No one knows what new order of society will come forth from the existing world chaos. Whatever it may be, men and women must be given opportunities to earn their daily bread in the production of things worth while, whether economically productive or desirable in a social sense. Now is the time, as proposed by Major Hallihan, to prepare plans in advance, rather than, by neglect in this respect, to invite a return of the soul-crushing, leaf-raking days of 1933.

The field for advance planning is well-nigh limitless. Many projects at once come to mind, such as water supply and sanitation; the control, purification and shore protection of streams; conservation of natural resources; light, heat, and power; housing and hospitals; parks and parkways and the beautification of the roadside; and transportation by rail, highway (including back roads), water, air, and pipe line. What a land this would be could a fraction of the money expended for destruction be devoted to construction of such a high order. Idealistic? Yes; but were people ever more in need of ideals to buoy their hopes for a better day? With it, however, must go the practical to save civilization when the present false prosperity ends.

It is not alone for the future postwar era that the planning of public works is needed. Right now much of it is required for wartime defense, as stated by Major Hallihan. Perhaps this is most true with the railroads. Deferred maintenance in that field gives ground for alarm; and, too, the lowest inventory ratio to operating revenue in thirty years, barring three, reported in the *Railway Age* of March 29, 1941, which in case of invasion or a similar emergency might spell disaster to the Nation.

Obsolescence of railroad equipment is marked, the Federal Reserve System's bulletin of October, 1940, reporting that in the period between 1930 and 1940 the percentage of locomotives less than ten years of age had dropped from 20 to 6, and of freight cars from 34 to 14. This condition is emphasized in the *Railway Age* of January 4, 1941, which, in prophesying car shortages, calls attention to the locomotive situation in which 68.5% in service in 1940 were twenty-one or more years old against 57% five years ago. Planning coupled with action in this field is needed immediately, not only for normal peacetime traffic, plus an evenly distributed comparatively small increase for defense traffic, but also for the many exigencies of war in case of home invasion or military operations beyond the borders in Canada or Mexico, or in more distant lands.

In their fixed properties the railroads also call for immediate planning and corresponding action in order that they may not obstruct the progress to be expected in equipment, service, and operating efficiency. The strengthening

[8] *The Wall Street Journal*, March 20, 1941.

and refinement of many thousands of miles of track, including bridges, the repair and modernization of yards, locomotive terminals, shops and shop tools, fuel and water facilities, work equipment, signals and interlocking, and freight and passenger stations; numerous other improvements such as by-passes around centers of congestion and the separation of grades; and the making good of deferred maintenance—all these call for several years of expenditures amounting to something like a billion dollars annually, in which is included provision for equipment. The aforementioned issue of the *Railway Age* goes into this question in great detail, as does its issue of January 7, 1939.[9]

The urgency of the need for doing something about the railroad situation is made startlingly apparent in the *Railway Age*, which states in its issue of January 11, 1941, that "to be frank, the financial results of the operation of the railways lend some color to the the charge of 'decadence,'" and that "the Nation cannot be restored to economic health and military invincibility if it continues to nurse along a transportation cancer within its economic body." That "military invincibility" in this respect is not to be expected from the railroads, unaided by federal intervention, is indicated by these quotations from an authoritative railroad source:

> "War participation * * * is outside of a consideration of the railroads' ability to carry on under conditions properly incident to that of National defense."
>
> "It would be unreasonable to anticipate the financing of equipment for such purposes from funds collected for the transportation of passengers and goods in a competetive transportation endeavor."
>
> "Enemy invasion of the United States * * * does not appear likely in the near future."
>
> "Plans on the part of the Government for activities under National Defense * * * have not heretofore been revealed to railroad managers."

In the light of these circumstances the value of public works on which Major Hallihan dwells would appear to be clearly evident; likewise, the wisdom (as he states under the heading "Possibilities of Public Works") of having "present an ample reservoir of projects that have gone through the time-consuming preliminaries of engineering examination and legal and financial authorization," coupled with prompt action in such instances as that of the railroads.

BERNARD L. WEINER,[10] M. AM. SOC. C. E. (by letter).[10a]—Considered merely as a tabulation of statistics, the author's data may probably be accepted as substantially correct—even though definitions of such terms as "national income" are not given. It is obvious that, unless care is taken, duplication may easily occur. For the purpose of this discussion, however, it is not necessary to go into refinements. It will be sufficient for the purpose even if the data are accepted as being merely indexes of relative values. In any case, it seems to the writer that the conclusions drawn from them are not justified. It is

[9] See also "Financial Requirements of Railways," Interstate Commerce Commission *Statement No. 3911*, March, 1939; and "Report of Committee Appointed September 20, 1938, by the President of the United States to Submit Recommendations Upon the General Transportation Situation," December 23, 1938.

[10] With Madigan-Hyland, Long Island City, N. Y.

[10a] Received by the Secretary April 2, 1941.

only necessary to plot the statistics to see that the conclusions as to cause and effect are unwarranted. Fig. 1 shows that, in relation to national income, the variation in public construction is so slight that it is unreasonable to suppose that the latter is the cause of the variation in the former. The author states that the 1938 drop in income is generally attributed to the reduction of federal aid in the latter half of the preceding year. The chart shows that, while

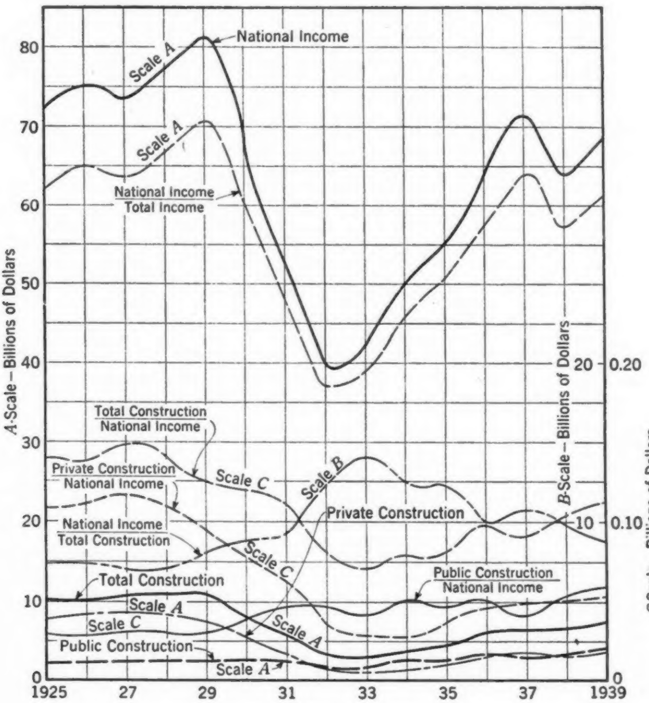

Fig. 1

there is a slight wiggle in the public construction curve, the total construction curve remains practically level. The ratio curves show further that, just prior to 1932, construction was decreasing faster than income and then increased slower than income. One is led to believe that construction and national income are both affected by factors other than those treated in this paper—factors that are of a magnitude comparable to the results they are supposed to control.

The author shows that in the ten-year period, 1929 to 1939, there was a loss of 53 billion dollars in private construction, whereas public works showed a gain of a mere four billions. During the same period, the loss in national income based on the 1929 income was about 250 billion. The four-billion dollar increase in public construction can hardly be credited with having more than a very minor effect on these large sums; the increase shrinks into in-significance by comparison. It might be argued that the 1929 income is not a just base for comparison since it is a maximum; but the fact remains, nevertheless, that the same industrial and agricultural capacity, as well as the manpower that was available and capable of producing the maximum amount in one year, had the same potential physical ability to produce a like amount in any other year. Whatever it was that prevented the national economy from functioning at the same level—or at even a higher level, as might reasonably be expected in a dynamic economy—was also responsible for the lag in construction. The "drop in the bucket" that constitutes public construction cannot, it seems to the writer, be the cause of the rise in the national income after 1933. The true controlling causes or factors, however, must be found; and it is perhaps no exaggeration to state that it constitutes the most important problem confronting the United States and the world today.

To declare that the most important problems that confront the nation are immediate recovery and long-range planning for the stabilization of industry is to state the obvious. The fact that the nation is now engaged in an undeclared war—or is in state of non-belligerent intervention, if this characterization is preferred—does not change matters. It is the opinion of the writer that the so-called domestic problem cannot be solved without taking the international situation into account and the two problems are so inextricably intertwined that they are really one and the same problem. Many panaceas have been offered for the solution of both problems treated separately, no two being alike. The conclusions drawn by the author are merely the echos of a political party pointing with pride to its supposed achievements—and at best, the paper is merely another panacea. The fact remains that the world is confronted with a failure of more than major importance—civilization is threatened. All methods from the appeal to the brotherhood of man to self-interest have been tried—and all have failed. In the domestic field, industry has been spending vast sums of money on physical research with the result that, as the products of industry approach perfection, the economic crisis becomes more and more acute. At the same time, wars are becoming ever more frequent and more destructive. On the other hand, except for such superficial studies as the charting of market trends in an effort to forecast the future by extrapolation, non-physical research has been neglected almost entirely.

In other words, the reason that the economic difficulties have not been solved is that no real honest attempt to do so has been made!

Studies, such as this one which the author has attempted, are necessary, and they should be made by engineers and scientists; but they are merely a "step in the right direction" toward the ultimate goal—a solution to the problem of war and poverty. That a solution is possible should be obvious. There is no reason to suppose that this problem with which the world is faced is somehow so peculiar that it will not yield to the same scientific treatment that has solved so many other problems. The same mind of man that is capable of producing the truly great marvels of modern civilization should also be capable of producing the solution for this non-physical problem. Engineers and scientists have been sadly remiss in their duties as citizens in this respect. They are both peculiarly fitted by the very nature of their training to tackle the economic problem.

A little consideration should show that the engineer in the rôle of economist is not at all a strange or far-fetched idea; in fact, it is very logical that he should be peculiarly fitted for it. What else is the work of the engineer—in the broad sense—but bringing order out of chaos? How else but by research to determine the facts, and by analysis to determine the solution from the facts, does the engineer work? Most important of all, the engineer has another qualification. It has been remarked often that the engineer acts for both his client, the owner, and for the contractor, and the disinterestedness of his judgment is never questioned. In other words, the engineer has the recognized distinction of being free from bias in forming his judgments. The very fact that those who have attempted to solve the economic problem have been led astray by their biased, preconceived ideas is one of the principal reasons why failure has always resulted. Those who succeed finally in finding a solution must be able to follow their studies through with scientific rigor, no matter where they lead—a specification that fits the engineer more than it does any other group. Finally, the same methods that made possible the physical basis of modern civilization should and could, logically, be applied to achieve the primary purpose of all human endeavor: To make the necessities and even the luxuries of life available to all mankind.

It might really be said that it is not so much a question of the engineer's becoming an economist, as it is one of applying engineering methods and procedure to economics.

Albert Ed. Scheible,[11] Assoc. M. Am. Soc. C. E. (by letter).[11a]—The subject is both interesting and timely. It is interesting because there has not been much published on the subject in the form in which it is presented in this paper; and it is timely because many persons are already beginning to think about the future—what will happen after the present conflagration has burned itself out. One evening the writer listened to an address by a noted sociologist on the subject, "After the War—What Then?" Engineers of every branch would do well to study this question. If the engineer does not do this, members of some other profession probably will. That is what happened during the past depression when many avoidable mistakes were made. Those who had instigated the program failed to realize that such an undertaking could be handled best by engineers.

It is the writer's opinion that here is a chance for professional and technical societies to take an active part in that "human engineering" which one hears so much about. Technical societies, working with the local professional groups, could do much to promote thought, and in some cases, could initiate a program of planning for the future.

As the author states, public works is not the entire answer to unemployment, but it can, and should, take the strain from other methods that will have to be adopted when a similar situation has to be met in the future. Based,

[11] Prin. Asst. Engr., R. F. MacDowell, Cleveland, Ohio.

[11a] Received by the Secretary April 3, 1941.

then, on the experience gained in the recent past, it should be possible to make many improvements. To that end the organized engineering profession should bend every effort.

Having been directly connected with non-federal projects, both as a government employee and later as an employee of a consulting engineer, the writer would like to present some views regarding the subject.

The hectic days at the beginning of the program in 1933 may be discounted. The present WPA program seems to have reached a point of more or less stability and, of course, PWA has ceased to function, at least in the matter of accepting new projects.

Those persons entirely unfit for manual labor, including the aged, should not be employed on construction projects. Some attempt has been made to employ only men who are physically fit. For some reason, the government rules applying to physical fitness have not been enforced as stringently as have some of the other rules, many of which are not nearly so important. More recently, in an apparent attempt to keep the WPA alive, this regulation seems to have been "winked at."

A planning program is of the utmost importance. Although the federal government "foots the bill," or most of it for labor at least, it should not expect financially harried municipal officials to "jump through the hoop" every time the unemployment curve takes a sharp rise. Most municipalities are always understaffed, particularly with respect to engineering personnel. Indeed, many small municipalities have no engineering departments, which may be one of the reasons for such poorly planned projects. The time to plan projects is now. As emphasized by the author, municipalities are becoming poorer for obvious reasons. They have no money with which to plan projects that may not be executed in the near future. The federal government should make funds available now, by grants to municipalities, for this work.

Under the present program (referring particularly to the WPA), it seems that all kinds of construction work, with the possible exception of paving, must be done in the winter time. An effort to find more protected work in the winter time would be very helpful. A program planned far enough in advance might aid in this respect, and perhaps a frank admission that most construction work, now being done or attempted, is out of place in the winter time. However, this reverts to the need for some other type of program during the winter months when the unemployment curve seems to rise. The writer believes that this frank admission should be made and that a new approach to the problem should be sought. The present program seems to have been started and then left hanging.

The contractor, it appears, is the forgotten man in the entire program. He did have some work under PWA but the program was so short that the average small contractor was lucky to secure one or two jobs, and at a ridiculously low sum. Every WPA form requires that the sponsor make a statement to the effect that the work proposed is not a normal function of the sponsoring body, and yet the writer has before him the report of a department of a certain state giving the amount of sewer and water work done with WPA aid during 1940, work that heretofore was done mostly by contract. If this condition continues, the contractor will soon take the place of the American Indian as the Vanishing American.

Due to rules and regulations, local offices must take a very arbitrary stand on all matters of policy. In the business world there is a saying, "the customer is always right." In the case of the WPA it is very often difficult to tell which is the customer. The local offices should have a certain degree of authority and there should be a certain degree of elasticity in the actual operation of a project. The writer hazards a guess that never has a construction project been executed in which everything was completed to the smallest detail as planned; and yet this is what is expected under the present system.

The need for the program is obvious. The administration is to be commended for its courage in initiating it. There is still much room for improvement. To that end every effort should be made by those on whom so much of the future will depend—the engineers.

H. B. Cooley,[12] Assoc. M. Am. Soc. C. E. (by letter).[12a]—Many of the difficulties that faced the execution of the public works program in the depression period are stated clearly by Major Hallihan. The writer feels that it is not too early to begin to lay the ground work and prepare for the inevitable adjustment period that is bound to occur with the cessation of hostilities and the armament program—whenever that may occur. Major Hallihan mentions some of the many difficulties that should be overcome and outlines remedial steps; but will the next occasion find the United States as unprepared as did the last period?

It should be remembered that the enlargement of public works as a method of alleviation of cyclical depression is merely one theory for the accomplishment of this purpose and is not "The" theory, as was stated in the "Synopsis" of the paper. Consequently, the advantages and disadvantages of the public works theory must be weighed against the advantages and disadvantages of various other theories. That there is any single remedy for our economic ills is doubtful and the increased expenditure for public works in times of decreased spending by private enterprise is undoubtedly an aid, but this increased spending should be accompanied by other measures that will supplement the program rather than work against it.

It seems to the writer that there are a number of reasons which may be given in justification of an expanded public works program in times of depression. By such a program is meant one that will take up the slack in normal construction—which norm would in itself be a varying term and would probably refer to the average volume of construction existing in a period of years immediately before the particular depression. Thus a program to relieve conditions in the depression of the 1930's would refer to the period of the late 1920's as normal, whereas a program in the 1940's would refer to the level established just before that period.

One justification for such a program might be the maintenance of the existing construction industry—the equipment, financial resources, workers

[12] Asst. Prof. of Economics and Business Administration, West Virginia Univ., Morgantown, W. Va.

[12a] Received by the Secretary April 11, 1941.

of all types, the suppliers of materials and equipment—in a condition as similar as possible to that existing before the depression. Such an objective would presuppose the use of existing methods of letting contracts and the use of existing organizations, contractors, equipment manufacturers and distributors, and material producers and dealers, wherever possible. This would assume costs on a comparable basis with the pre-depression costs, with, roughly, the same volume of business, the only differences being a shift from private construction to public construction and a change in emphasis from types of construction generally undertaken by private industry to types which could be undertaken by a public agency. In such a program, the flow of money through the various channels would be largely the same as the flow in the so-called "normal" period before the depression.

Another justification may be found by considering the increased public works as an offset to a decrease in private industrial activity. This would include not only normal construction activity but also the decline in manufacturing and distributing functions. If such a purpose is to be achieved the construction industry not only must be kept at its pre-depression level but it must be expanded to provide employment for those released in activities outside the construction industry and its related fields. This purpose may be accomplished by increasing the dollar volume of construction at the same unit costs, by maintaining the same dollar volume at decreased unit costs, or by altering the methods of construction so that a larger number of men may be employed on the same physical volume of work.

A third justification may be to find employment in the construction industry for all unemployed laborers who are capable of working and hence are employable. It seems to the writer that such a proposal is not feasible as many of the unemployed are not suited to construction activities. This applies to women, to the elderly worker, and to the majority of the so-called "white collar" workers, among others, although places can be found for some of these on construction projects. To attempt to place labor, which is not suited to such work, at work in a construction program must result in the creation of made work and "boon-doggling" projects, the value of which may well be questioned.

The values given in Table 2 indicate that the enlarged public works program failed to take up all the slack in the previous program when only the dollar expenditures are considered. Although public construction increased from an average expenditure in the pre-depression period of $2,318,000,000 to an average amount in what Major Hallihan terms the "post-depression" period of $3,268,000,000, there was actually a decrease in total expenditures from an average of $10,536,000,000 in the pre-depression period to $6,374,000,000 in the post-depression period, and from a high annual amount before the depression of $11,060,000,000 to a high annual amount after 1934 of $7,778,000,000. If the values are at all comparable it is apparent that the enlarged program of public works failed to meet the needs of the construction industry as it existed in pre-depression years and certainly could not have contributed toward the elimination of the slack in other industries not related to the construction industry. If costs were materially reduced per unit of physical work produced, due to a general decline in labor and material prices, then it is possible that the same physical volume was maintained even though the dollar expenditure was less. Such a decrease in costs, however, would have to be about 40% less than the pre-depression costs, based upon the total values.

Table 2 does show, however, that the average employment increased, as a result of the program, from 2,396,000 persons in 1928 to 3,493,000 persons in 1938, with a low of 1,001,000 in 1932. Was this increase of approximately 1,000,000 persons between 1928 and 1938 due to a change in methods of executing construction work or are the numbers comparable in any way? Due to the smaller number of hours worked per man each month, it is natural to suppose that the increase in the average number of men employed is due to this factor as much as to the increase in public expenditures from $2,484,000,000 in 1928 to $3,373,000,000 in 1938, especially when the decline in private construction from $8,576,000,000 in 1928 to $3,268,000,000 in 1938 is considered.

To judge the works program only on the basis of dollars expended or the number of men employed would seem to be fallacious. To what extent did it replace the existing industry of the 1920's; or did the shift from projects with a higher percentage of payment for material to the later projects with a higher percentage for labor tend to create other conditions which decreased the benefits flowing from the expenditure of the individual dollar in the first instance? In the former case the money was more widely circulated, and hence benefited more industries and more workmen, than was the result when a large part of the construction dollar was spent for construction labor and was immediately expended for consumption goods.

What was the ultimate value to society of the money spent on construction projects? Did the results obtained compare favorably or unfavorably with the results achieved in the preceding period in the 1920's? It seems to the writer that the cost of the works program should be evaluated from two viewpoints. One is the social view; and consideration must be given to the fact that large numbers of people were given employment who would otherwise have been idle.

The other consideration is the cost of the construction under such a program when compared to the cost of the same improvements by using, what might be termed for want of a better name, more orthodox methods. Men may be kept employed by having them spend their time on useless projects, "boondoggling," or they may be selected and employed so as to achieve the maximum efficiency for their labor without regard to the number employed. Also, an intermediate course may be pursued that will afford employment to larger numbers than in the second case but at the sacrifice of efficiency. The latter course was the one pursued and may have been the best course but it would seem desirable to know just what was achieved by such a program, both from the human viewpoint and also from a construction efficiency viewpoint, and just how much was sacrificed on the one hand in order to achieve the benefits on the other. Until this information is obtained, publicized, and discussed, it would seem that any plans based upon past experience rest upon a weak foundation. The two objectives are confused and merged into one another so that it is difficult to determine just what benefits have been derived from the program in the past.

To illustrate this confusion reference may be made again to the expenditure and employment values of Table 2. It was noted that the total employment increased by a yearly average of about 1,000,000 persons between 1928 and 1938, whereas the total expenditures for construction decreased $4,419,000,000 per year in the same period. Assuming that construction costs decreased during this period, that living costs also decreased and that, due to a change in types of projects and methods of executing the work, there was a larger percentage of the construction dollar paid to labor, what does this indicate? Was the individual workman benefited in proportion to the amount expended or were his annual earnings reduced? It may be that his standard of living was materially lowered because of a reduction in his actual annual wage. On the other hand, the steadiness of his employment throughout the year, even though for fewer days in the month, may have increased the average annual wages per worker. Which is correct and how does this benefit to the individual (after all there was a benefit to some when total unemployment was the alternative) compare with the cost to society for this social benefit to the human being and the physical benefit as measured by the actual value (not the cost) of the physical structures?

The situation may well be likened to that of a manufacturer producing a number of different products. His over-all operations result in a profit and for this reason he may be lulled into a feeling that all is well with the business. More detailed information, however, might reveal the fact that certain operations are unprofitable and that the entire enterprise would be benefited by discontinuing certain activities. Such an analysis might well be undertaken with relation to the public works program.

Philip W. Henry,[13] M. Am. Soc. C. E. (by letter).[13a]—Congratulations are due the author for presenting so clearly and concisely the expenditures made by the federal government during the recent depression in order to aid state, county, and municipal construction projects and to afford employment for those who were out of work. The details given in Tables 1 and 2 present a clear picture of the situation; and the inclusion in Table 2 of the national income for the years 1925 to 1939, inclusive, is pertinent. A statement of the yearly direct federal debt for the same period would also have been appropriate. Therefore, the writer has taken the liberty of including it in Table 3 for the purpose of comparison with the cost of public construction and national income, which are taken from Table 2.

From this statement it appears that in the depression period, when presumably governmental help was most needed, there was actually a slight decrease in the yearly average of public construction, notwithstanding an increase of 1.772 billion dollars (9.5%) in the federal debt, and a decrease of 25.1 billions (33.0%) in the national income.

[13] Cons. Engr., New York, N. Y.

[13a] Received by the Secretary April 16, 1941.

In the post-depression period there was an increase of 0.950 billion dollars (40.9%) in the yearly average of public construction compared with the pre-depression period, an increase of 16.661 billions (89.4%) in the federal debt, and a decrease of 11.4 billions (15.0%) in the national income. Evidently the

TABLE 3.—COMPARISON OF NATIONAL FINANCES, 1925 TO 1939

Year	Yearly Average, in Billions of Dollars					
	Public construction	Increase %	Federal debt [a]	Increase %	National income	Decrease %
Pre-depression, 1925–1929	2.318	0.0	18.641	0.0	76.0	0.0
Depression, 1930–1934	2.312	0.0	20.413	9.5	50.9	33.0
Post-depression, 1935–1939	3.268	40.9	35.302	89.4	64.6	15.0

[a] For fiscal years ending June 30; values are from the *Monthly Bulletin* of the National City Bank of New York for February, 1941.

federal government was slow in coming to the rescue, and its greatest expenditures, shown in public construction and increased debt, were in the post-depression period instead of during the depression itself. The author, noting this, states (see heading "Conclusion"): "To make a program of public works fully effective in assisting to take up the slack in industry, its full power must be thrown in promptly upon the inception of the depression."

Although no doubt most, if not all, of this public construction was well planned, well executed, of permanent value, and desired by the community, was it advisable for the federal government to assist state, county, or municipal projects by donating part of their cost? The local people know whether such projects—no matter how desirable—are really needed. If so, they should pay for them in full. The federal government could properly purchase local securities issued for construction (if sound) during the period of a reluctant bond market. Later on there would be no difficulty in selling them to the public, as has been demonstrated by the sales made in recent years by the Reconstruction Finance Corporation of such securities.

As far as public construction is concerned, the writer is not in accord with the author when he says that the "* * * heavy expenditures for direct relief and work expedients started the national income up from a low of 40 billion dollars in 1932 to 42.3 billions in 1933, and 50.1 billions in 1934." Table 2 shows a decrease in public construction from 1.878 billions in 1932 to 1.827 billions in 1933 and an increase to only 2.619 billions in 1934. More likely the increase in the national income was due to the recovery of the people from their state of panic that started in 1929. In the opinion of the writer the recent depression was similar to the major depressions of 1837, 1857, 1873, and 1893, all of which showed distinct recovery in six years or less with little help from the government.

Just what the federal government should do during major depressions is a moot question. In the depression of 1893 which, in the memory of the writer was just as serious as that of 1930–1934, the President of the United States took the position that it was not the function of the government to support the people, but that it was the function of the people to support the government. At any rate, the depression of 1893 was over in four years with an increase of only $262,336,000 in the federal debt (for the purchase of gold), whereas, in the depression of 1930–1934, it was increased from 16.185 billion dollars in the fiscal year, June 30, 1930, to 40.440 billions in 1939, a net increase of 24.255 billions. Therefore, this new method of dealing with depressions cost the people of the United States as much as did World War I, when the debt increased from 1.225 billions in 1916 to 25.482 billions in 1919, a net increase of 24.257 billions.

Not all, by any means, of this increased debt between 1930 and 1939, was due to public construction, for, although there was an increased debt from 1930 to 1939 of 24.255 billions, the author shows in Table 2 that the total excess of public construction during that period amounted to only 4.720 billions, or about 20%. However, the total excess of public construction for the post-depression years (1935–1939), which amounted to 4.750 billion dollars, was 35% of the increase in the federal debt of 13.387 billions, during that period, when the debt ascended from 27.053 billions in 1934 to 40.440 billions in 1939.

The writer has no complaint to make of the good intentions back of the public construction policy, and indeed back of all the other expenditures undertaken to counteract the effects of the depression; but he points out that the effect of any legislation is the same, regardless of the motives back of it. No individual can pursue a policy of borrowing continuously for ten years without experiencing the consequences, but his policy will affect only a few people; but when a government pursues such a policy, however, all the people are affected. Of course, in times of war greatly increased governmental expenditures are necessary, and have been incurred in all our wars, regardless of consequences; but, inasmuch as in previous depressions the country has recovered with little or no help from the government, the writer is not at all convinced that the increased expenditures for public construction, noted by the author, and other expenditures, represented by the increase of 24.255 billion dollars in the federal debt between 1930 and 1939, were justified. This seems particularly true when the similar depression of 1893 caused a rise in the public debt of only 1% of that amount.

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DISCUSSIONS

SIMPLIFIED THEORY OF THE SELF-ANCHORED SUSPENSION BRIDGE

Discussion

BY A. J. MEEHAN, M. AM. SOC. C. E.

A. J. MEEHAN,[10] M. AM. SOC. C. E. (by letter).[10a]—As much as the suspension bridge has occupied the spotlight during recent years, the self-anchored type has not enjoyed great prominence. The complexity of its analysis, which has existed heretofore, has been one of a number of contributory factors responsible for its infrequent selection. The long-awaited "Simplified Theory of the Self-Anchored Suspension Bridge" is now at the disposal of designers, for which warm thanks are due to Mr. Gronquist.

Possibly a discussion on this paper should be confined strictly to theory. However, taking a liberal viewpoint of "theory," it may be regarded as a cog in the production of something concrete or tangible. As such, the writer submits his generalizations, prompted by the experiences of his association in the rather complete preparation of preliminary designs for two separate river crossings by self-anchored suspension bridges. These preliminaries each consisted of a main span of 480 ft. Loaded side spans on one were 165 ft each, and flanking approach spans gave an aggregate length of 1,128 ft for this project. For the other layout, side spans of 150 ft were planned, to which were added a number of approach spans, resulting in a total bridge length of 1,590 ft. Each project provided a 26-ft roadway and two cantilevered sidewalks of 4-ft clear width.

In the self-anchored group of suspension bridges are those having a combination of counterweighted anchorage pier and self-anchored side span. At the sites previously mentioned, high water conditions prohibited this construction. During flood stages, the end piers, acting as gravity anchorages, would have been submerged to such an extent that the resulting buoyancy would have seriously impaired their usefulness.

Column action may require excessive sections in the side-span stiffening girder or truss to resist the horizontal component of the cable pull. In some

NOTE.—This paper by C. H. Gronquist, Assoc. M. Am. Soc. C. E., was published in February, 1941, *Proceedings*.

[10] Senior Bridge Engr., State Dept. of Public Works, Div. of Highways, Bridge Dept., Sacramento, Calif.

[10a] Received by the Secretary March 26, 1941.

stiffening trusses, the lower chord takes the horizontal cable reaction. In such cases, the top chord details may become awkward in order that a direct cable connection may be made with the bottom chord.

To transmit end-span loads ideally (including lateral forces, such as wind and earthquake), it is probably more important in this type, than in any other kind of structure, to adopt a compact and effective framing arrangement. Such a requirement gives stiffening girders the advantage of selection over stiffening trusses, due to a more unified action. It is probable that a steel floor might serve as the element to carry transverse shear by web action between stiffening girders acting as flanges.

In the projects with which the writer was connected, although trusses were considered, stiffening girders were given the "decision." The stiffening girders were cambered nearly sufficient to carry the entire dead-load moment. In other words, the stiffening-girder dead-load thrust practically coincided with the neutral axis. An effort was made to carry the remainder of the moment in the floor system. Difficulty was encountered in providing enough rivets and transmitting the loading from one floor panel to the next. Excessive material was required for shear development between the floor system and the concrete slab. However, the studies included steel-floor adaptation and also an attempt to relieve some of the stiffening-girder thrust with the lateral system.

Except for shorter spans, the satisfactory solution of the end connection, of either parallel wire cables or wire-rope strands, to the stiffening member can become as involved as any problem in structural steel detailing. Of course, the cables or chains could be connected to points other than at the extreme end of the stiffening girder, but this adds moment and shear due to the upward component of the cable reaction.

This type of bridge is obviously penalized from a cost standpoint by additional dead load due to details whose sole purpose is to counteract eccentricity or whose function is to restrain buckling tendencies of the stiffening girders acting as columns. Of further economic concern is the handicap involved in the provision for materials required for erection purposes only, which may include temporary connections or auxiliary and jacking struts or increases in sections of the permanent members due to the excess of erection stresses over normal. Some savings might be effected by scheduling the erection procedure so that use would be made in a temporary position—during the earlier stages of erection—of some of the materials required for other positions in the finished structure.

Eyebar chains are superior to cables because of easier connections at the stiffening member ends and simpler erection connections. At present (1941) eyebar production has been curtailed due to defense needs, so that all desirable sizes are not available; but at no time have eyebars been competitive bid items since the acceptable kind have been manufactured by only one company.

Falsework costs are greater in proportion for longer spans using trusses than in shorter spans using girders. Falsework for self-anchored suspension spans must remain in place not only during steel erection, but possibly until the concrete slab has been completed. This long period tends to increase the charge for the contractor's plant, which the owner pays indirectly. In streams where floods may develop during steel erection, the hazard is obvious.

Girders are generally more economical than trusses for the stiffening members in cases of reasonable span lengths and in ordinary roadways. To date, self-anchored spans have generally been short enough to permit girders to be used in lieu of stiffening trusses. However, recent tests have revealed that, from the standpoint of aerodynamic stability, holes should be provided in the tops and bottoms of girder webs up to 25% of the web area as one means of effecting such stability. Perhaps if such an area of open space is desirable through the sides of stiffening girders, one's preference may turn to stiffening trusses. It is recognized that the aerodynamic phenomenon does not demand attention when the span-to-width ratios are low as compared to the range of tried and proved structures.

Since simplification of the theory is the author's aim, with the ultimate objective of popularizing the type, it may be well to recognize some of the further obstacles to such progress. At present there is no accepted design specification for self-anchored suspension bridges. As a consequence, such development has stagnated. What, for instance, would be the acceptable live loading for a structure financed under such circumstances? Invariably what one gets when trying to secure an advance opinion (from an agency disbursing public bridge funds) is some arbitrary answer ranging up to 50% more than factual conditions may permit—more explicitly, say, a required value of 60 lb per sq ft live load instead of a submitted 40 lb per sq ft. Glenn B. Woodruff and Norman C. Raab,[11] Members, Am. Soc. C. E., have demonstrated the ultraconservativeness of such a requirement. They indicate that the maximum possible vehicular live load on a suspension bridge will be obtained with traffic at a standstill (hence, without impact), and with cars placed bumper to bumper, which condition produces a live load of 35 lb per sq ft. When the load is moving slowly, it may be only 25 lb per sq ft (impact to be added).

Another case in point concerns the cable. The effect of live loading per square foot may expand the safety factor tremendously. To produce the maximum stress in the main cable, the full length of the bridge is loaded. If a safety factor of three were required for the breaking strength of the cable when stressed under full dead and live load, and if the ratio of dead-to-live load were 3 to 1, then the safety factor for the live load would attain the unreasonable value of 9.

The unsatisfactory situation with respect to secret production details of heat-treated eyebars and of processing cable wire is alike, inasmuch as the designer openly admits his dependence upon the manufacturer. To protect himself, the designer resorts to specifying minimum acceptable materials and conditions. Although secretive, the manufacturers are nevertheless cooperative.

The variations in wire-rope strand construction make it difficult to obtain an accurate forecast of the resulting modulus of elasticity after the pre-stressing operation. However, if pre-stressing is adopted, one should require that the

[11] *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 614.

procedure develop a stress at least equal to the maximum design stress. The fullest advantage of pre-stressing will result if, as an added precautionary measure, it is specified that the ropes are to be socketed before pre-stressing.

It seems possible that, for certain locations, skewed suspension bridges are required, although they are never seen. In view of the present National Defense demands and the attendant difficulties in securing adequate quantities of structural steel, it seems likely that the concrete tower might find application in suspension-bridge design, plastic flow of concrete and other difficulties to the contrary notwithstanding.

An effort has been made herein to summarize some of the pitfalls that the enthusiast for self-anchored suspension bridges will encounter before his dream bridge can begin to function. Personal observation of the three self-anchored suspension bridges in Pittsburgh, Pa.—at Sixth, Seventh, and Ninth streets—has demonstrated amply the practicality of such structures to the writer. It is sincerely hoped that a newly rejuvenated era of self-anchored suspension bridges will be the response to the stimulating influence of Mr. Gronquist's illuminating contribution.

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DISCUSSIONS

ANALYSIS OF BUILDING FRAMES WITH SEMI-RIGID CONNECTIONS

Discussion

BY MAURICE P. VAN BUREN, ASSOC. M. AM. SOC. C. E.

MAURICE P. VAN BUREN,[4] ASSOC. M. AM. SOC. C. E. (by letter).[4a]—A basis for effecting valuable economies in certain types of structures is afforded by this interesting paper. However, one important correction to the analysis should be made. In comparing the moments at the face and at the center of a joint, consideration must be given to the reaction as a load on the structure. It does not appear that this has been included in the examples given. It can be provided for easily by the addition of a term to both Eq. 4*a* and Eq. 4*b*. At the point of inflection of the column, the reaction is uniformly distributed, and will cause a moment about the joint center of $V\,g\,b$, in which $g\,b$ is the distance from the joint center to the centroid of the half column. This moment is of opposite sense to the moments $V\,b$, and the last terms of Eqs. 4*a* and 4*b* will then read: $-\,V_{AB}'\,(1-g)\,b_{AB}$ and $V_{BA}'\,(1-g)\,b_{BA}$, respectively. The significance of this effect is apparent in the case of the flanged column illustrated in Fig. 5(*a*), for which g approaches 1, and the term involving V practically vanishes. The authors' comments on the extent to which final results will be affected is awaited with interest.

Corrections for *Transactions: Proceedings*, March, 1941, in Fig. 11, letter column tops "*a*," "*b*," and "*c*," and column bases "*d*," "*e*," and "*f*," respectively; in Fig. 12, number the joints, ground floor—6-7-8-7-6, second floor—3-4-5-4-3, and roof—1-2-1; on page 425 the abscissa in Fig. 13 should read "Ratio $\dfrac{\text{Column Width}}{\text{Beam Length}}$."

NOTE.—This paper by Bruce Johnston, Assoc. M. Am. Soc. C. E., and Edward H. Mount, Esq., was published in March, 1941, *Proceedings*.

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[4a] Received by the Secretary March 24, 1940.

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DISCUSSIONS

PLASTIC THEORY OF REINFORCED CONCRETE DESIGN

Discussion

BY MESSRS. ROBERTO CONTINI, AND A. A. EREMIN

ROBERTO CONTINI,[53] ESQ. (by letter).[53a]—Although the method of ultimate strength for purposes of design appears very logical and satisfactory, the developments for the determination of the strength of structural members seem to be rather uncertain. All the computations are based on the consideration that the equivalent rectangular stress diagram at failure must have a height equal to 0.85 f_c' in order for the center of gravity of that rectangle to coincide with the center of gravity of the actual stress diagram.

This develops, it seems, from the shape of the stress-strain diagram for concrete and from the fact that the maximum bending moment is developed when the maximum concrete strain is about twice the one causing the maximum stress. However, as the author himself states (see paragraphs following Eq. 3), "This ratio will vary with the quality of the concrete and the steel strain." As the height of the rectangular equivalent diagram is certainly correlated with that ratio, the value of 0.85 f_c' appears questionable.

Moreover, even if that value is satisfactory for concrete at failure, it could be erroneous, to some extent, in case of under-reinforced beams. In this case, in fact, the full concrete strength is not yet developed when the steel yields. The concrete stress diagram has a different shape than the one at maximum concrete strength, and its center of gravity is in a different position, so that the lever arm of the steel is different.

The case of under-reinforced beams is to be given consideration, because very often the size of a beam is controlled by a large bending moment confined to only a part of it, or by conditions other than the maximum bending moment. When this occurs, it would be wasteful to provide the full percentage of reinforcement required to balance the section, at least for some parts of the beam.

The writer thinks, however, that it is possible to avoid the use of uncertain

NOTE.—This paper by Charles S. Whitney, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by Messrs. L. E. Grinter, and Basil Sourochnikoff; March, 1941, by Messrs. R. W. Stewart, George C. Ernst, Homer M. Hadley, and Robert W. Abbett; and April, 1941, by Messrs. Paul Andersen, and R. A. Caughey.

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[53a] Received by the Secretary February 18, 1941.

values such as 0.85 f_c', and to proceed, instead, with the analysis of reinforced concrete members by simple considerations very similar in principle to the ones used in the usual theory of reinforced concrete. Such analysis, outlined hereafter, leads to the plotting of diagrams that are very easy to use.

The study is limited to simple bending of rectangular and T-beams, with tensile reinforcement only; but, if the system should prove satisfactory, it should not be difficult to extend it to any type of load and reinforcement. The following assumptions are made: (*a*) The strain distribution is linear through the entire depth of the beam; (*b*) the tensile action of concrete is negligible; (*c*) the stress-strain characteristics for concrete are well known and can be represented by means of a curve such as Fig. 3; and (*d*) the steel follows Hooke's law to the yield point—that is, to failure.

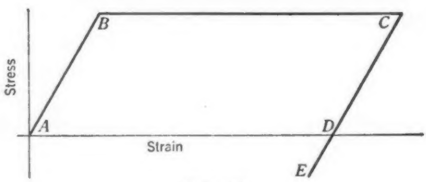

Fig. 15

The first assumptions are made also by the author. In order to justify assumption (*d*) the writer refers to a paper by J. A. Van den Broek, M. Am. Soc. C. E., in which a stress-strain diagram[54] for structural steel is plotted as shown in Fig. 15. Professor Van den Broek states:

> "Although the curve [Fig. 15] may appear idealized, this is not necessarily the case to any great degree. To be sure, stress-strain curves frequently are recorded with a bulge instead of the sharp angle shown at Point *B* * * *. It must be noted that the things tested are specimens and not filaments. For example, an ordinary steel bar suffers a differential stress between material in the outside and the center of the bar due to the cooling of the core (on the cooling bed) after the outside of the bar is already black and hardened. Furthermore, the rate of running the testing machine affects the curve. In other words, variations from stress-strain curves of mild steel, as shown in the figure, may frequently be assigned to the test specimen rather than to the material. In several hundred tests reported in 1918[55] the writer found the horizontal part of the curve very pronounced and coincident with the proportional limit."

It must be noted that Professor Van den Broek refers to tests made by strain control method, which is the one used for determining the stress-strain curve for concrete given by the author.

Rectangular Beams.—On these assumptions the following relations can be written (using the author's notation): From the linear strain relation—

$$\epsilon' = \frac{1 - k}{k}\,\epsilon \qquad \text{(44a)}$$

Because of Hooke's law for steel—

$$T = A_s f_s = A_s\,\epsilon'\,E \qquad \text{(44b)}$$

in which E = Young's modulus up to yield point. Since $T = C = A\,f_c'\,k\,b\,d$:

$$A_s\,\epsilon'\,E = A\,f_c'\,k\,b\,d \qquad \text{(44c)}$$

[54] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Vol. 105 (1940), Fig. 2, p. 640.

[55] "The Effect of Cold Working on the Elastic Properties of Steel," Carnegie Scholarship *Memoirs*, Iron and Steel Inst., Vol. IX, 1918; see, also, *Engineering*, July, 1918.

Eliminating ϵ' from Eq. 44c by means of Eq. 44a:

$$A_s \frac{1 - k}{k} \epsilon E = A f_c' k b d \qquad (45a)$$

Therefore,

$$k^2 + k \frac{A_s \epsilon E}{A f_c' b d} - \frac{A_s \epsilon E}{A f_c' b d} = 0 \qquad (45b)$$

and

$$k^2 + k \frac{p \epsilon E}{A f_c'} - \frac{p \epsilon E}{A f_c'} = 0 \qquad (45c)$$

from which, by solving for k and taking the positive root:

$$k = -\frac{p \epsilon E}{2 A f_c'} + \sqrt{\left(\frac{p \epsilon E}{2 A f_c'}\right)^2 + \frac{p \epsilon E}{A f_c'}} \qquad (46)$$

For any value of ϵ, the corresponding value of A can be selected from Fig. 3(b). Therefore, everything being known concerning the right side of Eq. 46, it will be possible to plot a series of curves k against ϵ for different values of p

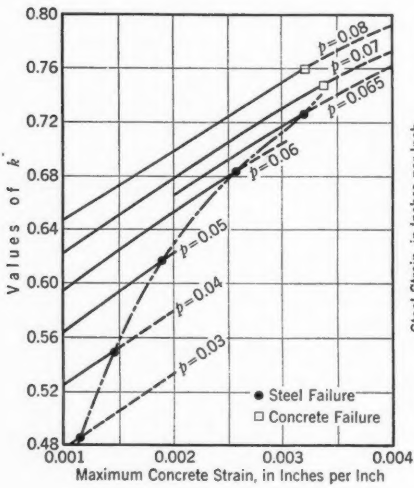

FIG. 16

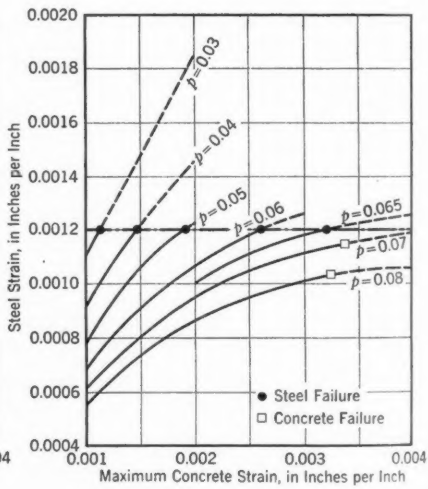

FIG. 17

(see Fig. 16) and, by using Eq. 44a, to plot ϵ' against ϵ. All the elements are now available to plot $\frac{M}{f_c' b d^2}$ against ϵ by using Eq. 2. The curves given in Fig. 17 were plotted by using the following characteristics of the materials: (a) For concrete, the curves given by the author, in Figs. 3(b) and 3(c) with $f_c' = 4{,}000$ lb per sq in. (The limit of $\bar{x}$ when ϵ approaches zero should be 0.667.) (b) For steel, $E = 30{,}000{,}000$ lb per sq in. and $f_s' =$ yield point stress $= 36{,}000$ lb per sq in.; consequently, $\epsilon_y' =$ strain at which yield begins $= 0.0012$.

The curves were plotted as if the steel followed Hooke's law indefinitely; therefore they are of no significance beyond the point at which the steel strain reaches the value $\epsilon_y' = 0.0012$ (dotted parts of curves). As shown by Figs. 17 and 18, for small values of p the beam fails by tension in the steel, because the steel strain reaches the value ϵ_y' before the full strength of the concrete is developed. If p is high, the maximum resisting moment, as controlled by concrete, is reached when the steel strain is still less than ϵ_y' and the beam fails by compression in concrete.

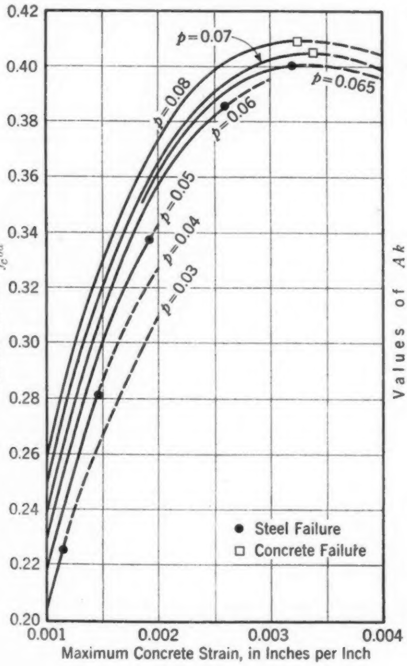

FIG. 18

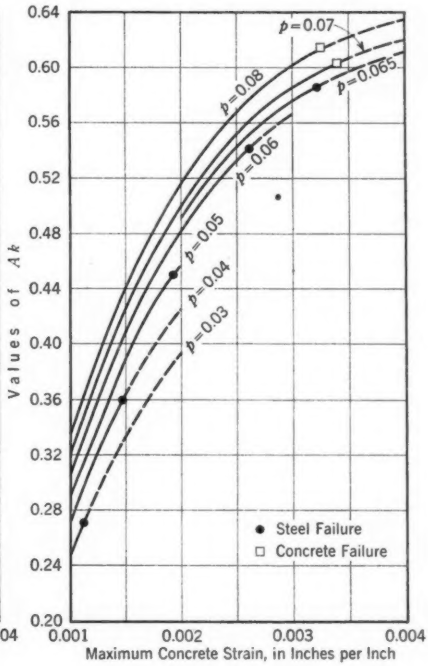

FIG. 19

A "balanced" design is obtained when the maximum resisting moment, as controlled by concrete, is reached at the same time that the steel strain reaches the value ϵ_y'. At that point the beam is on the verge of failure both by compression in concrete and by tension in steel.

For the selected characteristics of the materials, the value of p for balanced design happens to be a little more than 0.065. For under-reinforced beams the value of ϵ, corresponding to ϵ_y', can be found also (in addition to the foregoing) by plotting the curve $A\,k$ (Fig. 19) and noting that $T_{max} = \epsilon_y' E A_s$. This quantity is known; and, as $T = C$, one can write: $\epsilon_y' E A_s = A f_c' k b d$; or,

$$\frac{\epsilon_y' E p}{f_c'} = A\,k \qquad (47)$$

in which the left term is known. The value of ϵ, corresponding to the value of $A\,k$ found by means of Eq. 47, is read from the curves.

In Fig. 20 the value of the maximum resisting moment is plotted against p. This, therefore, is the fundamental diagram for design purposes. Also the value of k at failure is plotted.

T-*Beams.*—For T-beams with tensile reinforcement (using the notation of Fig. 21), if $k < t$, the formulas found for rectangular beams apply as well.

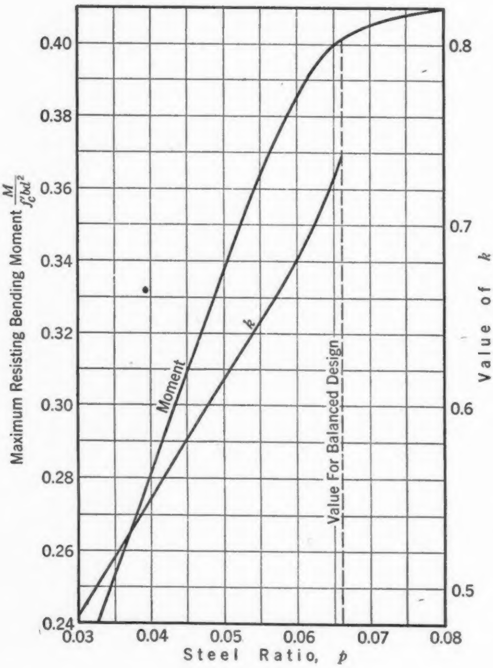

FIG. 20

The author states that, in his theory, formulas for rectangular beams apply directly to T-beams if the compression depth a is less than the thickness of the flange. It seems, however, that in any case the length $k\,d$ should be considered instead of a; otherwise it becomes implicitly assumed that the stress distribution is rectangular, which, on the contrary, is sharply denied by the author.

If $k > t$, the total compressive force will be the same as for a rectangular beam of width b, less the part corresponding to the shaded area in Fig. 21(a). If: $\frac{b - b'}{b} = s$; C_1 = compressive force for rectangular beam of width b; C_2 = compressive force corresponding to the shaded area, to be subtracted; and $C = C_1 - C_2$ = effective compressive force; then:

$$C_1 = A\, f_c'\, k\, b\, d \qquad (48a)$$

$$C_2 = A_2 f_c' (k - t)\, b\, d\, s \qquad (48b)$$

and

$$C = C_1 - C_2 = f_c'\, b\, d\, [A\, k - A_2 (k - t)\, s] \qquad (48c)$$

Area A is read from Fig. 3(b) in relation to ϵ, maximum concrete strain, and A_2 is read in relation to ϵ_2, the concrete strain at the lower face of the flange. The linear strain distribution yields the relation:

$$\epsilon_2 = \frac{k - t}{k}\,\epsilon \qquad (49)$$

Assume that: $\bar{x}_1\, k\, d$ = distance of the line of action of C_1 from the neutral axis; and $\bar{x}_2\, k\, d$ = distance of the line of action of C_2 from the neutral axis.

The moment of $C = C_1 - C_2$ about the neutral axis will then be:

$$(C_1 - C_2)\,\bar{x}\,k\,d = C_1\,\bar{x}_1\,k\,d - C_2\,x_2\,(k - t)\,d \qquad (50)$$

in which $\bar{x}\,k\,d$ is the distance of the line of action of C from the neutral axis. Solving for $\bar{x}$:

$$\bar{x} = \frac{C_1\,\bar{x}_1 - C_2\,\bar{x}_2\left(1 - \frac{t}{k}\right)}{C_1 - C_2} \qquad (51)$$

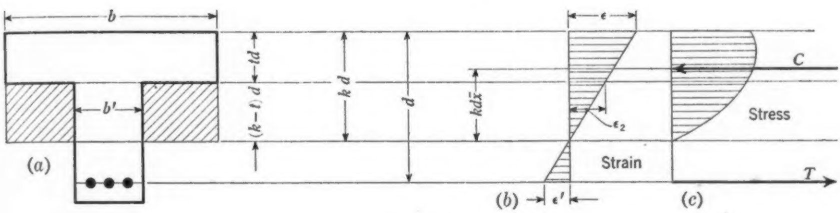

Fig. 21

The total bending moment is:

$$M = C\,d\,(1 - k + k\,\bar{x}) \qquad (52)$$

and, substituting for C the value found by Eq. 48c:

$$M = f_c'\,b\,d^2\,[A\,k - A_2\,(k - t)\,s]\,(1 - k + k\,\bar{x});$$

or,

$$\frac{M}{f_c'\,b\,d^2} = [A\,k - A_2\,(k - t)\,s]\,(1 - k + k\,\bar{x}) \qquad (53)$$

Furthermore, since $\epsilon' = \frac{1 - k}{k}\,\epsilon$ and $T = A_s\,f_s = A_s\,\epsilon'\,E = C$:

$$A_s\,\epsilon'\,E = f_c'\,b\,d\,[A\,k - A_2\,(k - t)\,s] = A_s\,\epsilon\,E\,\frac{1 - k}{k} \qquad (54)$$

That is:

$$\frac{p\,\epsilon\,E\,(1 - k)}{f_c'\,k} = A\,k - A_2\,(k - t)\,s \qquad (55a)$$

or

$$k^2\,(A - A_2\,s) + k\left(\frac{p\,\epsilon\,E}{f_c'} + A_2\,s\,t\right) - \frac{p\,\epsilon\,E}{f_c'} = 0 \qquad (55b)$$

If $s = 0$, Eq. 55b becomes the formula for rectangular beams; if $s = 1$ the action of the compressed part of the web is neglected.

Eq. 55b is not as simple as the one for rectangular beams, and actually is not solvable for k by simply assigning values to ϵ, as in the case of rectangular beams, because A_2 is unknown. This quantity, in effect, is read from Fig. 3(b), the determination of which, by means of Eq. 49, requires a knowledge of k. Therefore Eq. 55b can be solved either by trial or graphically. In this latter case, for each value of ϵ, A_2 should be plotted against k both by means of Eq. 55b

and by means of Eq. 49 and Fig. 3(*b*). The intersection of the two curves gives the value of k corresponding to the inherent value of ϵ.

Once k has been plotted against ϵ for given values of t and s and for a series of values of p, all the elements are available for plotting the following curves (all with ϵ as abscissas): ϵ_2, by means of Eq. 49; A_2, from Fig. 3(*b*); $\bar{x}$, from Eq. 51 after having determined $\bar{x}_1$ and $\bar{x}_2$ from Fig. 3(*b*); and finally, for determining the maximum resisting moment as controlled by concrete $\frac{M}{f_c' \, b \, d^2}$ by means of Eq. 53.

For steel failure ϵ' will be plotted against ϵ, by means of Eq. 44*a*, and the designer can proceed in the determination of the resisting bending moment as he did for rectangular beams.

All these curves are plotted for a series of values of p. Finally, the resisting moment can be plotted against the steel ratio p; and, here again, this will be the fundamental curve for purposes of design.

In conclusion, sets of diagrams should be plotted for each combination of steel and concrete; in the case of T-beams, a set of diagrams would be necessary for each combination of t and s. As a matter of course, the suggested method should be tested with experimental results. The writer has attempted to show that, for purposes of design, it is not necessary to rely on formulas based on more or less empirical and uncertain data, when it is possible to devise diagrams for easy application, based on simple and fundamental considerations. In this sense, the discussion is an attempt to support and improve the arguments presented by Mr. Whitney.

A. A. Eremin,[56] Assoc. M. Am. Soc. C. E. (by letter).[56a]—It is well known that the modulus of elasticity for compression in concrete varies with its stress, age, and physical qualities. Attempts were made in this paper to develop the equations for stresses, considering variation of the modulus of elasticity.

Interesting equations were developed by Fritz J. von Emperger in 1936 for a beam with a rectangular section.[57] He considered the variation of the modulus of elasticity in terms of the ratio "n," which he computed for the maximum stresses in concrete and steel, and the percentage of reinforcement in the section. Mr. von Emperger expressed variation of stresses in the rectangular section by a trapezoid with the maximum stresses at the top of beam section continued to a depth of $\frac{2}{3}\,a$ and varied according to a straight line from the maximum stresses to zero stresses at the neutral axis, a being a distance from the top of beam to the neutral axis (Fig. 22). Mr. von Emperger has shown that, in a reinforced concrete beam with light reinforcement, a variation of the ratio "n" has a slight effect on the stresses. Furthermore, like the author, he reached the conclusion that varying the ratio "n" and the percentage of reinforcement in a beam section made it possible to increase the maximum stresses in concrete and steel.

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[56a] Received by the Secretary April 7, 1941.

[57] "Der Beiwert 'n,'" by Fritz J. von Emperger, *Beton und Eisen*, October, 1936, p. 324.

In his formula for stresses (Eq. 3), instead of varying the ratio "*n*," Mr. Whitney assumed the maximum strains in concrete and steel. Furthermore he assumed that the stresses in the rectangular section vary as a rectangular prism with the total resulting stresses concentrated in center. Evidently the

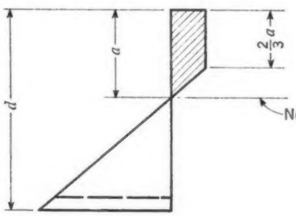

Fig. 22.—Distribution of Stresses

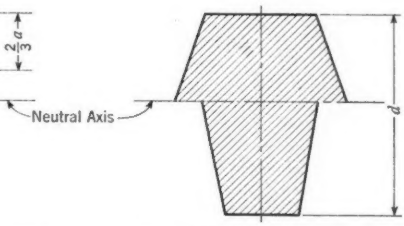

Fig. 23.—Equivalent Section

resulting equations may produce an economic design of reinforced concrete beams with small ratio of depth to span length. However, deep girders designed according to the author's formulas will require an excessive percentage of reinforcement.

To insure further accuracy in the equations for stresses in reinforced concrete beams, the influence of the section width should be taken into account. J. Melan has shown that the distribution of stresses in deep sections is not the same as in wide sections.[58]

Interesting equations for computation of stresses in concrete beams were developed by Rudolf Kern.[59] Like the author, Mr. Kern based his equation on the stress and strain curve obtained from a cylinder test. However, instead of assuming a rectangular prism distribution of stresses, Mr. Kern computed an equivalent trapezoidal section for the rectangular section of a beam, Fig. 23. Stresses in the equivalent trapezoidal section of a beam may be computed with standard formulas based on a straight-line variation. It is interesting to note that the equivalent beam sections determined by Mr. Kern may be used also for the computation of the elastic properties in rigid frames. Mr. Kern applied his equations in the computation of stresses in reinforced concrete arch ribs.

The equations of this paper should be modified to make them strictly applicable to the analysis of stresses in rigid frames or arches.

[58] *Beton und Eisen*, 1936, p. 315.

[59] *Loc. cit.*, January, 1930, p. 29.